flomdemping

flomvern og

flomhandtering

Thomas Skaugen

NVE

Estimating the mean areal snow water equivalent from satellite images and snow pillows

F

HYDRA - et forskningsprogram om flom

HYDRA er et forskningsprogram om flom initiert av Norges vassdrags- og energidirektorat (NVE) i 1995. Programmet har en tidsramme på 3 år, med avslutning medio 1999, og en kostnadsramme på ca. 18 mill. kroner. HYDRA er i hovedsak finansiert av Olje- og energidepartementet.

Arbeidshypotesen til HYDRA er at summen av alle menneskelige påvirkninger i form av arealbruk, reguleringer, forbygningsarbeider m.m. kan ha økt risikoen for flom.

Målgruppen for HYDRA er statlige og kommunale myndigheter, forsikringsbransjen, utdannings- og forskningsinstitusjoner og andre institusjoner. Nedenfor gis en oversikt over fagfelt/tema som blir berørt i HYDRA:

- Naturgrunnlag og arealbruk
- Skaderisikoanalyse
- Tettsteder
- Miljøvirkninger av flom og flomforebyggende tiltak
- Flomdemping, flomvern og flomhandtering
- Databaser og GIS
- Modellutvikling

Sentrale aktører i HYDRA er; Det norske meteorologiske institutt (DNMI), Glommens og Laagens Brukseierforening (GLB), Jordforsk, Norges geologiske undersøkelse (NGU), Norges Landbrukshøgskole (NLH), Norges teknisk-naturvitenskapelige universitet (NTNU), Norges vassdrags- og energidirektorat (NVE), Norsk institutt for jord- og skogkartlegging (NIJOS), Norsk institutt for vannforskning (NIVA), SINTEF, Stiftelsen for Naturforskning og Kulturminne-forskning (NINA/NIKU) og universitetene i Oslo og Bergen.

HYDRA - a research programme on floods

HYDRA is a research programme on floods initiated by the Norwegian Water Resources and Energy Directorate (NVE) in 1995. The programme has a time frame of 3 years, terminating in 1999, and with an economic framework of NOK 18 million. HYDRA is largely financed by the Ministry of Petroleum and Energy.

The working hypotesis for HYDRA is that the sum of all human impacts in the form of land use, regulation, flood protection etc., can have increased the risk of floods.

HYDRA is aimed at state and municipal authorities, insurance companies, educational and research institutions, and other organization.
An overview of the scientific content in HYDRA is:

- Natural resources and land use
- Risk analysis
- Urban areas
- Flood reduction, flood protection and flood management
- Databases and GIS
- Environmental consequences of floods and flood prevention measures
- Modelling

Central institutions in the HYDRA programme are; The Norwegian Meteorological Institute (DNMI), The Glommens and Laagens Water Management Association (GLB), Centre of Soil and Environmental Research (Jordforsk), The Norwegian Geological Survey (NGU), The Agriculture University of Norway (NLH), The Norwegian University of Science and Technology (NTNU), The Norwegian Water Resources and Energy Directorate (NVE), The Norwegian Institute of Land Inventory (NIJOS), The Norwegian Institute for Water Research (NIVA), The Foundation for Scientific and Industrial Research at the Norwegian Institute of Technology (SINTEF), The Norwegian Institute for Nature and Cultural Heritage Research (NINA/NIKU) and the Universities of Oslo and Bergen.

Estimating the mean areal snow water equivalent from satellite images and snow pillows

by

Thomas Skaugen
Norwegian Water Resources and Energy Directorate (NVE)

Preface

This report develops and investigates techniques to estimate the snow water equivalent using information from satellite images, and is part of the subproject F6, flood reduction, flood protection and flood management, under the HYDRA-programme. The purpose of this investigation has been to tie snow coverage, which routinely are monitored for flood forecasting purposes, to estimates of the snow volume. The report shows that additional information from snow pillows, which gives the frequency of snowfall events in the accumulation period, is crucial for the proposed method of estimation.

I would like to acknowledge Hans Christian Udnæs, who prepared the satellite scenes, and Dan Lundquist, who provided data and thoughtful comments.

Oslo, October 1998

Thomas Skaugen
researcher/project manager

Forord

Denne rapporten søker å utvikle metodikk for å estimere snøens vannekvivalent ved hjelp av satellitt bilder og er en delrapport under F6 innenfor flomdempning, flomvern og flomhåndteringsdelen av HYDRA-programmet. Hensikten har vært å forsøke å knytte snødekningsgrad, som rutinemessig innhentes til flomvarslingsformål, til volumestimater av snø. Rapporten viser at tilleggsinformasjon fra snøputer som viser hvor hyppig det har forekommet nedbør i snøakkumuleringsperioden er sentralt for den esimeringsmetodikk som er utviklet.

Personer som har bidratt med ideer og data er nevnt i under "Acknowledgements" bakerst i rapporten.

Oslo, oktober 1998

Thomas Skaugen
forsker/prosjektleder

Contents

Summary

By modelling the snow accumulation process in time and space as sums of random gamma distributed variables, the mean areal snow water equivalent (SWE) can be estimated. In the methodology we make use of the fact that sums of gamma distributed variables with a certain set of parameters also are gamma distributed variables with parameters being functions of the original and the number of summations. The measured snow/SWE at a point at a certain time t, can thus be seen as the accumulation, or the sum of the snowfall process from the beginning of the snowfall season up to t. The integration of these points, which give an area, can be seen as another summation. From snow pillows and precipitation gauges the value of daily accumulated precipitation/snow has been found to be well represented by a two parameter gamma distribution. This distribution has been found to be representative for large areas. The number of events where the precipitation was accumulated can be estimated from snow pillows situated in the area. The mean snow coverage over an area, which represents the summation of the individual points over an area, can be derived from satellite images represented in a GIS. The methodology is tested for two nested catchments of size 4723 km^2 and 19832 km^2 in a mountainous area in Southern Norway for eight satellite scenes. The results are compared with simulated snow reservoirs using a rainfall-runoff model, and found to agree well. Large discrepancies in the snow reservoirs between the proposed method and the rainfall runoff model are found in late spring and are probably due to errors in the estimated mean snow coverage.

1. Introduction

Severe spring floods in Norway are usually caused by a combination of intense snowmelt and precipitation. We can identify three independent climatic factors which unfavourable constellations can cause severe flooding: the snow pack, temperature and precipitation. Forecasting runoff based on these variables involves different time scales of prediction. Precipitation and temperature can only be forecasted on a short time scale, but an estimate of the snow pack at the time of melting results from monitoring the snow pack throughout the season of snow accumulation. Thus, to better be able to forecast the spring floods we need means of estimating the snow reservoirs at the onstart of the melting season. Estimation of the snow pack is made difficult by the extreme variability which is found when measuring snow, due to redistribution by the wind which again is affected by topographical and landscape features (Rango, 1996). Hydropower companies and the national flood forecasting service in Norway are using snow courses, snow pillows, simulation by rainfall-runoff models and satellite images to estimate the snow pack. Satellite images can, by the combined use of GIS (geographical information system) and DEM (digital elevation model), place the bulk of snow at a certain elevation in the catchment, which, combined with the knowledge of how the temperature develops with elevation can give important information on when the most intense discharge due to snowmelt can be expected. However, the current methodology is not yet able to determine the mean SWE (snow water equivalent) in a catchment from information provided by satellite images, thus in this respect the use of satellite images is yet of more qualitative than quantitative interest.

This paper intends to develop means to estimate the mean SWE over an area, by using information from satellite images. In order to do this we have to understand how the snow coverage is related to the snow volume, a problem that suffers from lack of relevant data and underdeveloped theory. Häggström (1994) compared the snow coverage derived from the NOAA satellite with the snow magazine simulated by the HBV-model (a rainfall-runoff model (Bergstrøm, 1992)) and found poor correlations. This result may imply that knowledge only of the snow coverage is not sufficient to assess the snow volume. In this paper the relationship between snow coverage and snow pack will be further investigated and three theoretical models will be proposed. The models differ in the way information other than the snow coverage is taken into account. Of the presented theoretical models for estimating the mean SWE over an area, we will concentrate on the method of modelling the areal snow accumulation as sums of gamma distributed variables. The results of this method will be validated against simulations made by the HBV-model and estimates based on snow courses.

Section two presents three theoretical models for relating the snow coverage to the snow volume. The data and study area is presented in section three, while a discussion of the assumptions concerning the method of gamma sums is in section four. Results and validation of the method of the gamma sums is provided and discussed in section five, and conclusions are found in section six.

2. Methodology

2.1. Linear relation between fraction of area over a threshold and the mean snow depth

It is known from studies of the estimation of mean areal rainfall from radar images that there exists a linear relation ship between the mean precipitation depth within an area, and the fraction of the area covered by precipitation over a certain threshold. This relationship has been developed into a method of estimation, the so-called threshold method, and has been subject to empirical and theoretical studies (Braud et al., 1993; Kedem et al, 1990). The relationship has the form of (Braud et al., 1993):

$$m(R(A)) = S(\tau)m(I(A,\tau)) + d(\tau) \quad (1)$$

where $m(R(A))$ is the mean rainrate over the area A, $S(\tau)$ is the slope of the linear relationship, $m(I(A,\tau))$ is the mean fractional area with a rainrate higher than τ, and $d(\tau)$ is the intercept. This method is ideal for remote sensing devices in that the spatial coverage of rainfall can be obtained for large areas at small time scales.

2.1.1 Discussion of the method

For obvious reasons it is tempting to apply the same methodology for the estimation of the mean areal snow depth. If the mean areal snow depth is a linear function of the fraction of the area covered by snow (similar to (1), with $\tau = 0$), then from a satellite image where the fraction of the area covered by snow could be determined, the mean areal snow depth could be derived. We will in this section discuss the potential of this method related to the estimation of the mean areal snow depth/mean areal SWE, and give an explanation on why such a linear relationship can not be, and that the empirical findings from the rainfall studies are the results of working on the linear part of the tail of an exponentially decreasing distribution function.

That a linear relationship is at best a simplification, can easily be seen from (1), where for any intercept values different from zero, the equation does not make sense.

E.g. we cannot have a mean areal rain rate different from zero $(m(R(A)) > 0)$ if the mean fractional area is equal to zero $(m(I(A,\tau)) = 0)$. In Figure 1, we have plotted the fraction of snow courses (data is presented below in section 3) for values greater than certain threshold values, and fitted linear regression lines. Similar to the rainfall studies, the relationship between the fraction of the snow course being over a certain threshold and the mean snow depth can very well be represented by a straight line for different thresholds (τ). However, if a distribution function is assumed for the areal snow depth (any exponential decaying distribution will do), then the fractional areas, can be considered as experimental outcomes of this distribution function. When the mean snow depth is plotted as a function of different thresholds, we see that although linearity may appear as a good approximation for a certain window of mean values, the approximation breaks down when this window becomes too large (see Figure 2).

The fractional area for the threshold zero, which is of interest when the task is to estimate the mean areal snow depth from satellite images, is not defined for the distribution function in Figure 2. However, when empirical values from the snow courses was assessed (see Figure 1), one can see that the threshold, $\tau = 0\ cm$ has the greatest variability with respect to predicting the mean areal snow depth. Further investigation of this method was not performed.

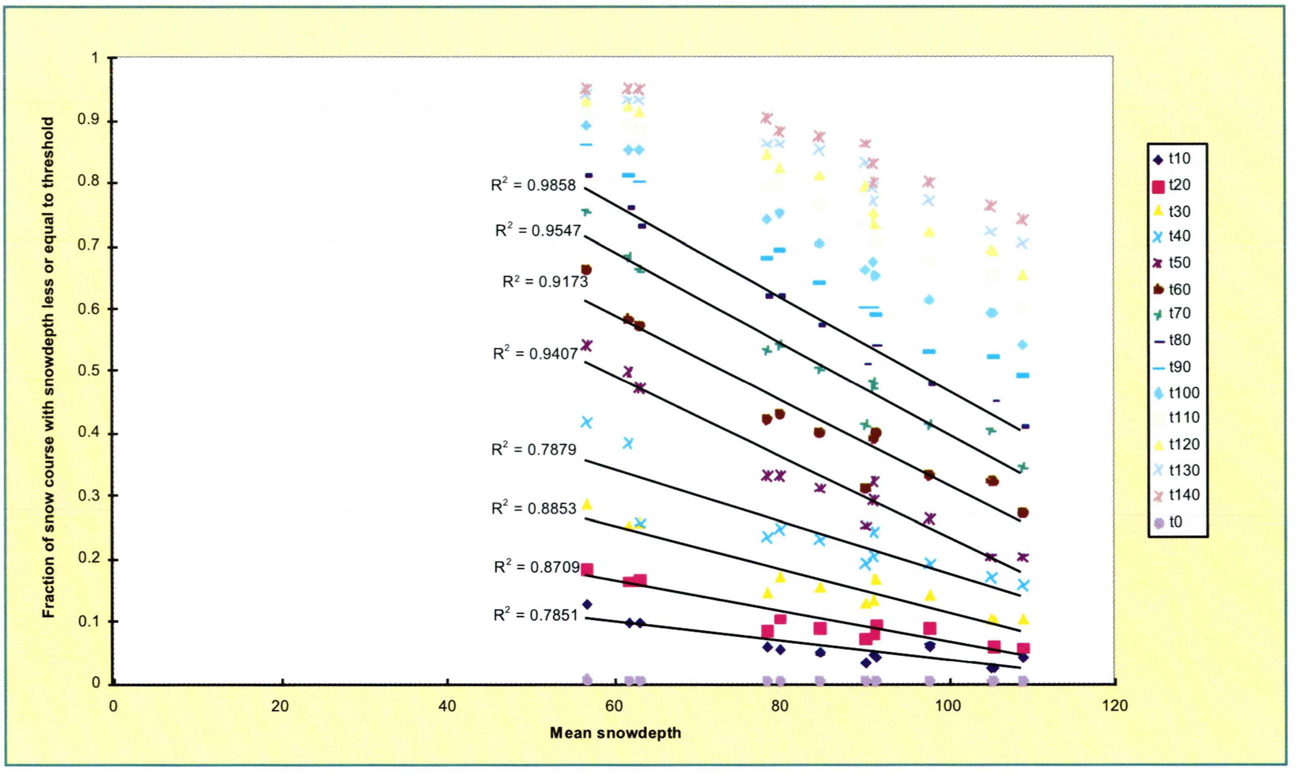


Figure 1. *Linear regression lines fitted to fractional areas less or equal to threshold of snowcourses. We note that threshold zero (t0) is uninformative in explaining the mean snowdepth.*

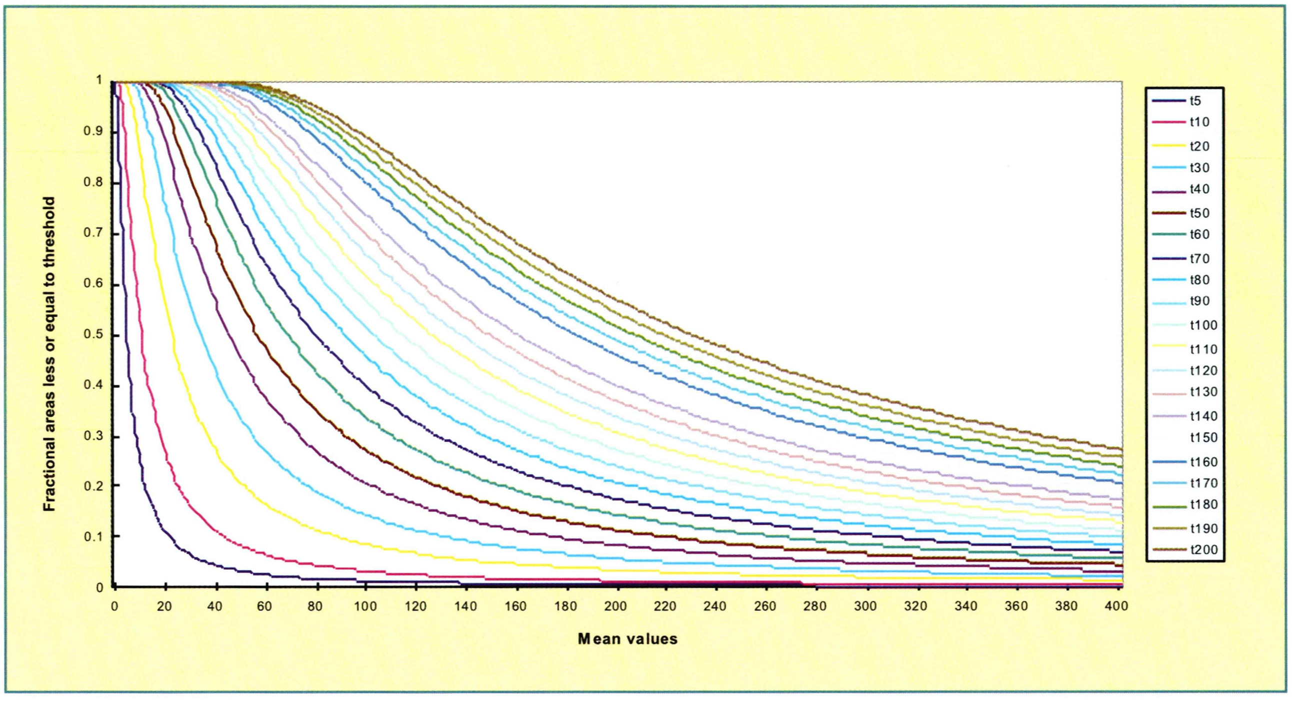


Figure 2. *An exponentially decaying distribution (Weibull) with the threshold as argument, and parameters determined from the mean.*

2.2. Identical relations in distribution functions

Let us assume that snow depth *(sd)* and snow coverage (sc) are two continuos variates with known probability density functions such that *sd* results from the transformation of sc:

$$sd = k(sc) \qquad (2)$$

where k is an unknown function and is strictly increasing. If the cumulative distribution functions (CDF) of *sc* and *sd* are denoted H*(sd)* and G*(sc)* then the function *k* can be defined as (Gottschalk, 1995 p.36):

$$k(sc) = H^{-1}(G(sc)) \qquad (3)$$

where H^{-1} is the inverse of the CDF H*(sd)*.

From studying the distribution of the pixels of snow coverages, *sc* (8 images are investigated) we find that the Weibull distribution (Haan, 1971, p. 114) with CDF:

$$G(sc) = 1 - Exp(-(sc/c)^{\beta}), \qquad c,\beta > 0 \qquad (4)$$

and probability density function (PDF):

$$g(sc) = \alpha / c\left(\frac{sc}{c}\right)^{\beta-1} Exp(-(sc/c)^{\beta}) \qquad (5)$$

where *c* is the scale parameter and β is the shape parameter, can be fitted (see Figure 3, a,b). If we further assume that a Weibull distribution also can represent the distribution of snow depth, an assumption which is justified by the fact that the gamma distribution, which is used in the next section for SWE, is very similar to the Weibull distribution, the transformation function of (3) can be derived from:

$$H(sd) = 1 - Exp(-(sd/a)^{\alpha}), \qquad a,\alpha > 0 \qquad (6)$$

The inverse can be expressed as:

$$sd = a(-\log(1 - H(sd))^{1/\alpha} \qquad (7)$$

and by inserting (4) as the argument in (7), we find the transformation function (see also Krzysztofowicz, 1997):

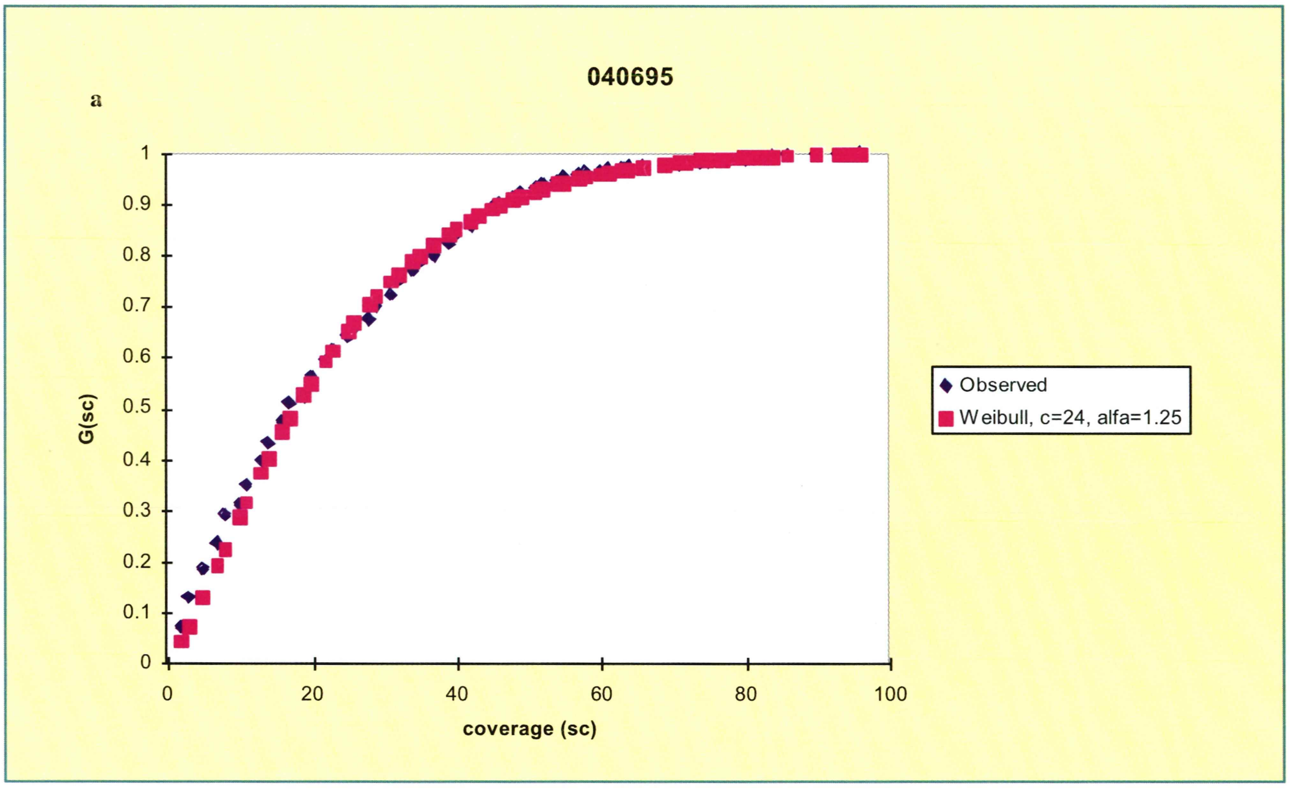
$$sd = a(sc/c)^{\beta/\alpha} \qquad (8)$$

The values of *c* and β can be estimated by fitting the distribution to the data of the satellite image at hand . Figure 3 (a,b) shows some typical distributions of snow coverage *(sc)* for different dates (see section 3 for a presentation of the data of satellite images).

We can state some properties of α. If α were equal to β, it would imply that snow depth was a linear function of coverage, *sc*, only scaled by the constant *a/c*. However α, has to be parameter less or equal to β because the accumulation of snow is observed to be a clustered process, which implies that a small increase in snow coverage thus may bring a large increase in snow depth.

Equation (8) appears to be simple, but with unknown α and *a*, the expression is underdetermined. If two sets of mean snow depth with corresponding coverages were known, and an assumption of a unique one-to-one correspondence of snow depth and coverage was justified, then (8) is solved. This information is not available and this method was not further pursued.

What the two methods presented in section 2.1 and 2.2 demonstrate is that for the estimation of the mean areal snow depth/mean areal SWE knowledge of just the coverage is not sufficient. The method proposed in the next section takes this into account and includes information on the frequency of precipitation accumulation events.



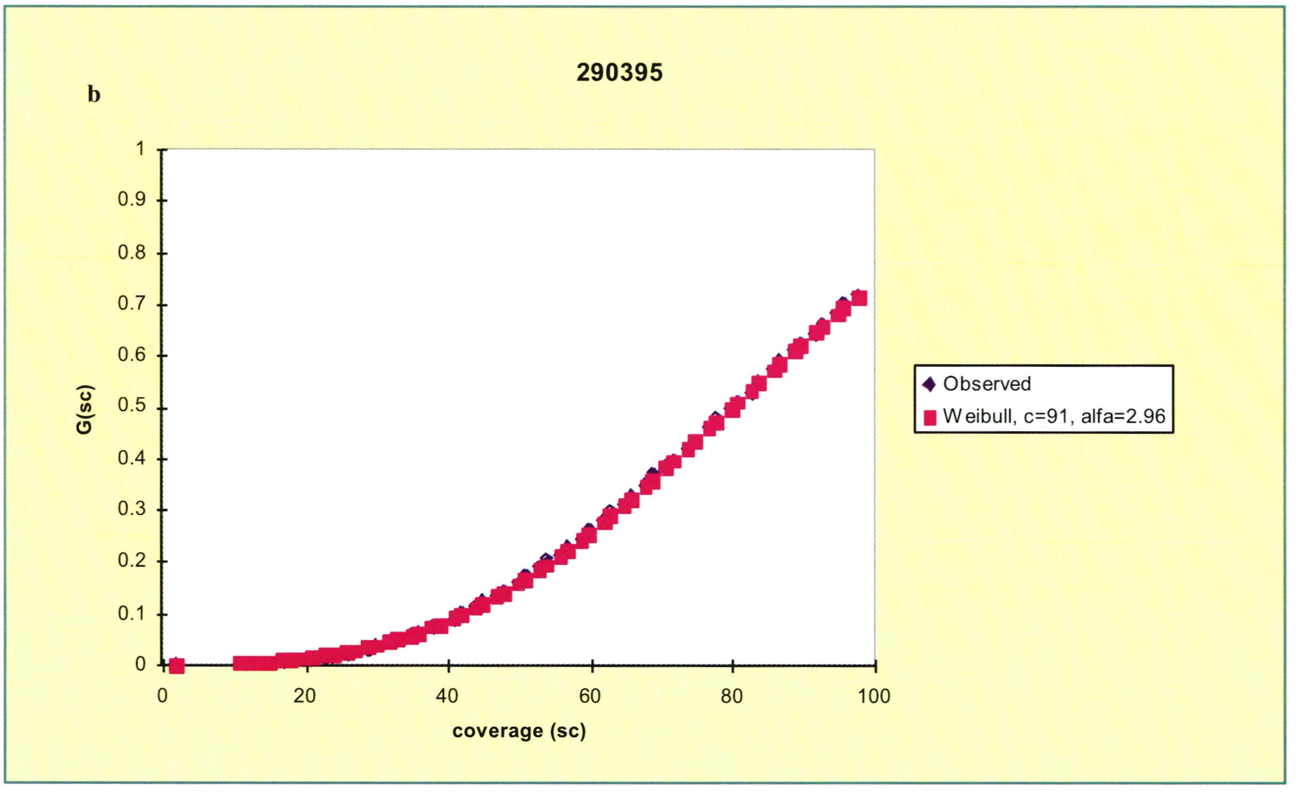


Figure 3. *Experimental and fitted (Weibull) probability distributions of coverage derived from satellite images. a) 040695 and b) 290395.*

2.3 Modelling the snow accumulation process as sums of gamma distributed random variables

The understanding of snow depth and snow cover, measured at a certain time t, as the sum of preceding events, is fundamental to the proposed method of estimating the areal snow water equivalent (SWE). If we consider y_i (x), in the following denoted y_i to be the SWE for snow fall event i at position x, then at time t after the n'th event we measure the sum Z_t (x), of the y's at point x:

$$Z_t(x) = y_1 + y_2 + \dots + y_n \qquad (9)$$

The volume of SWE over an area A is then the integral of Z over the area:

$$V_{t,A} = \int_A Z_t(x)dx \qquad (10)$$

or the discrete sum:

$$V_{t,A} = \sum_{k=1}^{K} Z_{t,k} \qquad (11)$$

and the mean areal value of the SWE is:

$$SWE_{t,A} = \frac{V_{t,A}}{A} \qquad (12)$$

Let y in (9) be a gamma distributed random variable with PDF:

$$f_{\alpha,\nu}(y) = \frac{1}{\Gamma(\nu)} \alpha^{\nu} y^{\nu-1} e^{-\alpha y} \qquad \alpha, \nu, y > 0 \qquad (13)$$

were α and β are parameters. The mean equals $E(y)=\nu/\alpha$ and the variance is equal to $Var(y)=\nu/\alpha^2$.

If the variables y are independent gamma variables, then $Z_t(x)$ in (9) is distributed as a gamma variable with parameters α and $n\nu$ (Feller, 1971, p.47). It can be noted here that for sufficiently large n, the distribution of $Z_t(x)$ will approach a normal distribution (Yevjevich, 1982, p. 144). The mean is equal to $E(Z_t(x))=\nu/\alpha$ and the variance is equal to $Var(Z_t(x))=n\nu/\alpha^2$. Now, according to (10), the volume over an area is the integral, which, of course, also is a sum. If the different $Z_t(x)$ are independent, then the volume of SWE, $V_{t,A}$, is also distributed as a gamma variable with parameters α and $Sn\nu$, where S is defined as:

$$S = \int_A I(x)dx \qquad (14)$$

where $I(x)$ is the indicator function defined as: $I(x)=1_{Zt(x)>0}$.

The mean and the variance of V , are respectively: $E(V_{t,A})=Sn\nu/\alpha$ and $Var(V_{t,A})=Sn\nu/\alpha^2$. If we let $a=S/A$ be the fraction of A where snow has been recorded , the mean areal SWE can be computed as:

$$E(SWE_{t,A}) = an\nu / \alpha \qquad (15)$$

3. The data and study area

This study is conducted on data from the Glomma basin, which is situated in South central Norway (see Figure 4). This basin is chosen because it is the largest in Norway (approx. 42 500 km^2), the hydropower interests in the area are significant and floods have a severe impact on the economy and the infrastructure. The basin is also relatively well instrumented with respect to hydro-meteorological variables. During the 1995 flood in June, the economical loss was estimated to be 1600 million Norwegian crowns (approx. 160 million pounds sterling) (NOU, 1996, p.43). The flood was caused by a combination of intense snowmelt and precipitation.

Four sources of data have been used in order to verify assumptions and estimate parameters in the method described in section 2.3: i) Snow courses, ii) precipitation gauges, iii) snow pillows and iv) satellite images. The data used will be further described below.

A snow course is a (often straight) transect, where the snow depth is measured for every 50 or 100 meters. From a mountainous sub catchment in the Glomma basin, (Fundin 258 km^2, elevation 1000-1600 m.a.s.l., see Figure 4) snow course data have been sampled in early spring (March / April) for 15 years (1980-1994). The length of the transects is about 50 measuring points. The water management association operating the Glomma reservoirs (Glommens og Laagens Brukseierforening, GLB), which conducts the data collection, has tried to use the same snow course transects so the data can be compared from year to year. Some studies have been conducted regarding the optimal way to perform snow course sampling. Gottschalk and Jutman (1979) suggest the following : i) Sampling with at least 50 to 100 meters interval, in both forest and open field, is advisable in order to avoid redundant information. ii) In order to double the precision of the mean, roughly five times the number of sampling points is needed and iii) snow course as a straight line or as a circle gives the smallest standard error.

The precipitation gauges are situated in the area from which we are going to estimate the mean SWE (see Figure 4). An important source of error, is precipitation loss due to wind. A recent report by co-operating Nordic meteorological institutes concludes that snowfall loss due to wind, is, dependent on the force of the wind, 0 - 70 % (Førland et al. 1996). The values, which are analysed here, are not corrected for wind loss and are provided from the Norwegian Meteorological Institute.

The snow pillows are considered to be point observations, but measure, in fact, over an area of 3.14 m^2 . The snow pillow provides daily observations of the accumulated SWE. An important source of error is the redistribution of weight due to the formation of snow crust above the snow pillow from periods of melting. Data from snowpillows are registered from the first snow fall, usually the beginning of October until the end of the melting season, usually the end of May, beginning of June.

We have 8 satellite scenes covering the Glomma basin. Images from the NOAA (National Oceanic and Atmospheric Administration) satellite with the AVHRR (Advanced Very High Resolution Radiometer) instrument are processed and presented in a GIS (Arc/Info), where snow coverage is given in % for each pixel of size 1.1 x 1.1 km^2. (Further information concerning details of the satellite and its operation can be found in Schjødt-Osmo and Engeset, 1997.) The images are from approximately the same hour in the day, ensuring similar light conditions for the different scenes. Because of problems with clouds, we settled for studying the distribution of snow coverage for two nested subcatchments of the Glomma basin, C1 of 4723 km^2 and C2 of 19832 km^2 (see Figure 4.). The satellite scenes were "cut" accordingly. The sources of error involved in using the satellite images should be stressed here. The albedo, which is fundamental for the reflectance of snow, is found to be decreasing as the snow layer grows older (Bras, 1990, p.263), which will lead to an underestimation of the snow coverage. Also shadows that appears in the images (caused by the angle of the sun) should give a systematic underestimation of the reflectance and thus the snow coverage. Forested areas in the two subcatchments also pose a problem. In spring these areas appear in the images as free of snow, while substantial amounts of snow can be found in these areas. The fraction of forested areas in the two subcatchments is found to be 44 and 36 % for C1 and C2 respectively.

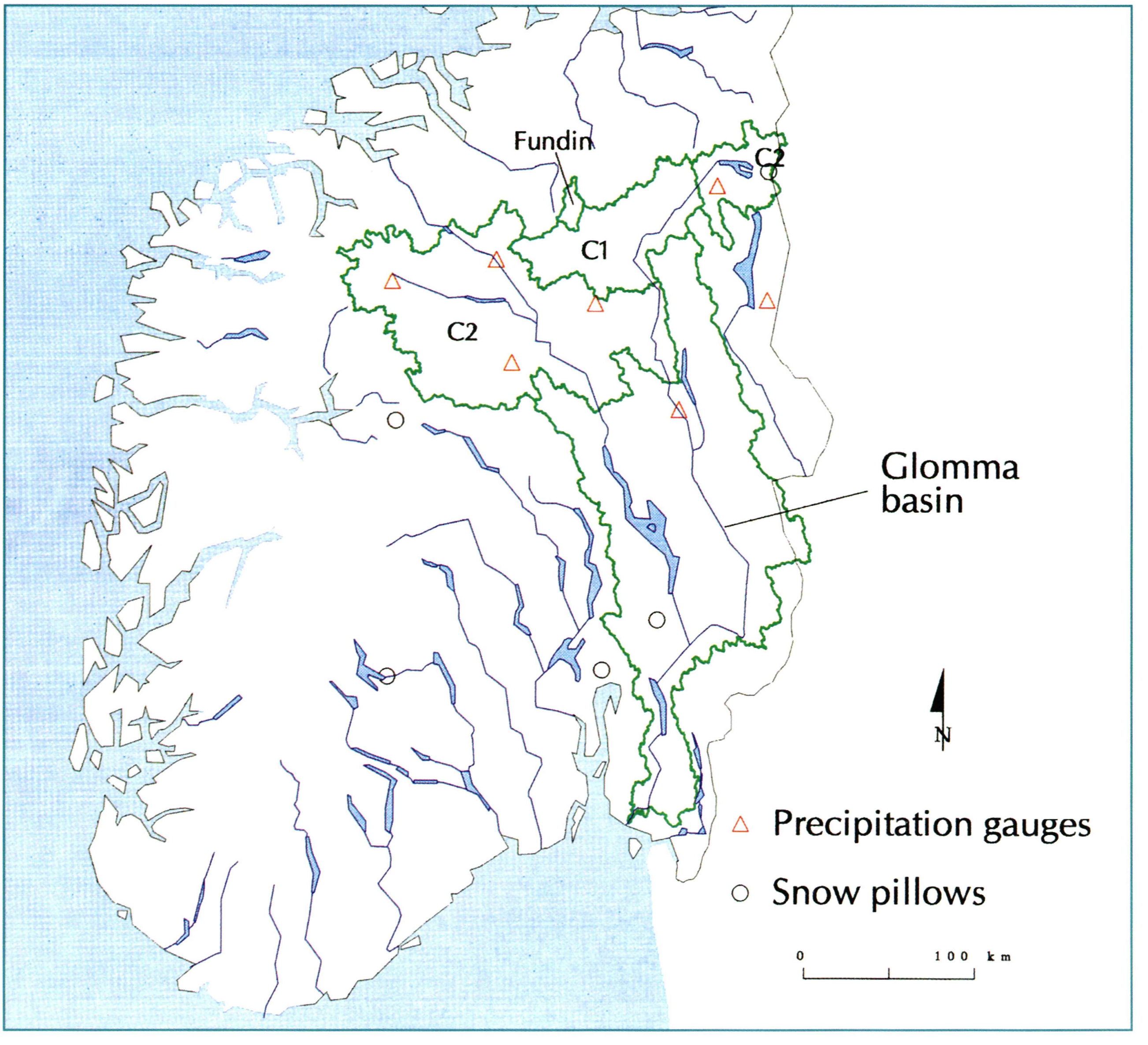


Figure 4. *The study area. The test catchments C1 and C2 are situated in the north part of the Glomma basin. C1 and Fundin are nested within C2.*

4. Justifying assumptions and estimating parameters

The data described in section 3 will be used to verify assumptions made in the formulation of the model presented in section 2.3. The assumptions that have to be investigated are summarised in the following:

i) The parameters α and ν are associated with properties of single snowfall events observed at points. For the methodology described above, these parameters are assumed to represent point behaviour everywhere in the considered area A. This implies that we assume snowfall to be a stationary process in time and space.

ii) The variable n describes the number of events with snowfall y, whose sum constitutes Z. This variable obviously has areal properties, in that snowfall events do not occur at singular points, but have an areal extension. The spatial magnitude of the events are unknown, but according to the methodology above, the events are assumed to be influential everywhere where snow is recorded at time t, i.e. S.

iii) The variable a is clearly connected to the spatial properties of snowfall. a is the fractional area where snow is recorded at the time t. The satellite images provides information on the percentage snow coverage for each pixel. The variable of interest is SWE, which, of course, has the same areal distribution as snow itself.

4.1 Stationarity and independence of snowfall in time and space

For the verification of the point properties of snowfall, we used data from five snow pillows and seven precipitation gauges situated in Southern Norway (see Figure 4).

Both daily accumulation and melting were analysed. The time series of daily values were separated into daily accumulation and melting values. Table 1 provides information on the different snow pillows and the estimated values of parameters α and ν. The parameters are estimated by the maximum likelihood method (Haan, 1977, p. 103).

Table 1. *Height above sea level (h.a.s.l), period of observations (October-June each year), the daily mean and parameters of the gamma distribution for accumulation and melting for the five snow pillows.*

Name	H.a.s.l. [m]	Period of observation	E(SWE) $_{[mm]}$ accumulation	α, accumulation	ν, accumulation	E(SWE) $_{[mm]}$ melting	α, melting	ν, melting
Vauldalen	820	1984-1997	3.55	0.46	1.61	10.44	0.12	1.23
Filefjell	950	1967-1997	3.93	0.35	1.38	10.03	0.08	0.83
Lybekkbråten	200	1983-1994	4.07	0.34	1.39	6.72	0.17	1.16
Brunkollen	370	1983-1994	4.71	0.26	1.23	10.17	0.10	1.0
Groset	950	1971-1994	4.72	0.30	1.39	8.76	0.12	1.06

Table 2. *Height above sea level (h.a.s.l), period of observations, the daily mean and parameters of the gamma distribution for snowfall for the seven precipitation stations. Only events where precipitation greater than 1, observed at temperatures below zero, are in the computation.*

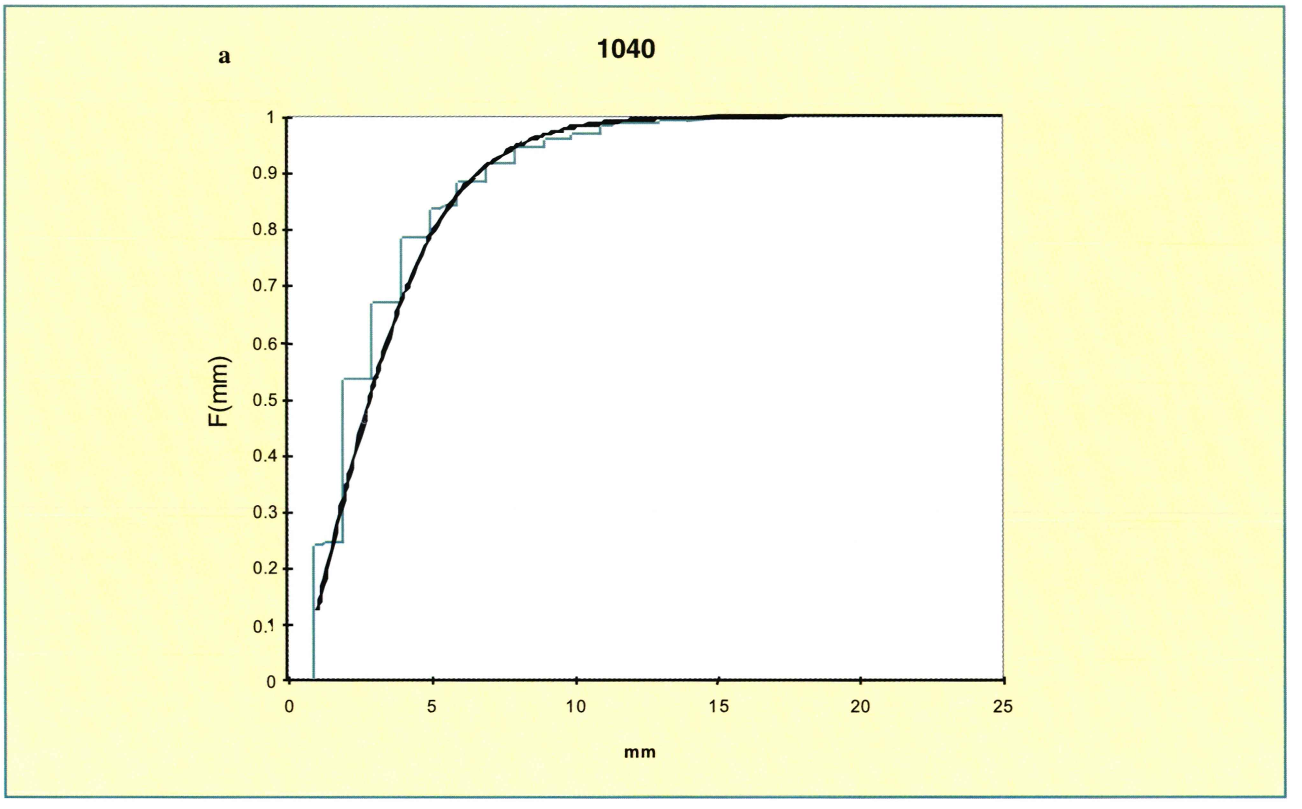
Station no.	H.a.s.l. [m]	Period of observation	E(SWE) [mm]	α	ν
1040	628	1978-1997	3.39	0.57	1.93
871	739	1978-1997	3.79	0.48	1.81
813	255	1978-1997	4.82	0.31	1.63
700	672	1969-1988	3.30	0.58	1.36
1367	865	1982-1997	4.15	0.50	2.10
1572	712	1970-1989	5.11	0.27	1.91
1674	626	1981-1997	3.79	0.43	1.50

Time series of different length of snowfall occurring when temperature is below zero are analysed. Table 2 provides information on the precipitation stations and the estimated values of parameters and. The parameters of the gamma distribution are estimated by the maximum likelihood method (Haan, 1977, p. 103).

When we compare the parameters for snow pillows and precipitation gauges, and take into account the different sources of errors associated with these values, we find that an assumption of stationarity in the studied area with respect to the parameters of the gamma distribution is justified. That the gamma distribution is a reasonable choice for snowfall can be verified from Figure 5 (a, b), which show fitted distributions to the snow pillow of Vauldalen, and the precipitation gauge no. 1040. Another indication to that the gamma distribution is suitable for modelling the snowfall process and its accumulation can be found when studying snow courses. For illustration, we have joined the different snow courses for a season in a joint sample, and estimated the parameters for the gamma distribution for two seasons (1991 and 1985). Figure 6 shows that the snow depth can be modelled as a summation of gamma distributed variables. The parameter α is identical for the two seasons (α= 0.027, this value is computed from snow depth in cm, and is not comparable with the one computed for SWE below), and the difference is found in the shape parameter vn. If v is considered constant and unique for the process, then the difference between the two seasons is the number of accumulations (n in the model formulation). In Figure 6, v is chosen to be v=0.0252 which corresponds to n=100 for the season of 1985 and n=63 for the season of 1991. We see a possible development of the snow cover for i=1, i=20 and i=n.

An assumption of stationarity in time can be justified from the investigation of the variability in precipitation from year to year. Table 3 shows how the daily mean of positive precipitation varies for the precipitation gauges and snow pillows for the seasons 1987/88 to 1995/96. The observed variability can partly be the result of poor sample size (the precipitation data especially suffers from this), but overall, the values are found to be in agreement with an assumption of stationarity in time and space. It can also be seen that, although not directly comparable, the values for the snow pillows and the precipitation gauges agree quite well.

In literature we can find references on the spatial dependence of measured snow depth *(sd)*. The method described above has as an assumption that $Z(x)$, which is the water equivalent for *sd*, for different positions x are independent and can be summed. Faanes and Kolberg (1996) conducted analysis of snow course data, and found that the autocorrelation dropped to approximately 0.2 for measuring points 50 metres apart, and 0.12 for points 100 metres apart, which coincides with the study of Gottschalk and Jutman (1979). When we take into account that the sampling space can be considered to be the resolution of the satellite image (1.1 x 1.1 km^2), the two above analyses suggest that the assumption of spatial independence is justified. It has also been claimed that topography has a fundamental influence on the distribution of snowfall, which renders an approach of assumed homogeneity in space meaningless. A recent study on the topographical influences on the mountainous snow depths found correlation coefficients in the range from 0.004-0.05 between snow depths and meters above sea-level (R^2=0.0058), curvature indices (R^2=0.0039) and exposition to westerly winds (R^2=0.05) (Faanes and Kolberg, 1996). One can conclude from this study that modelling snowfall using topography to explain the variability will not be very successful.



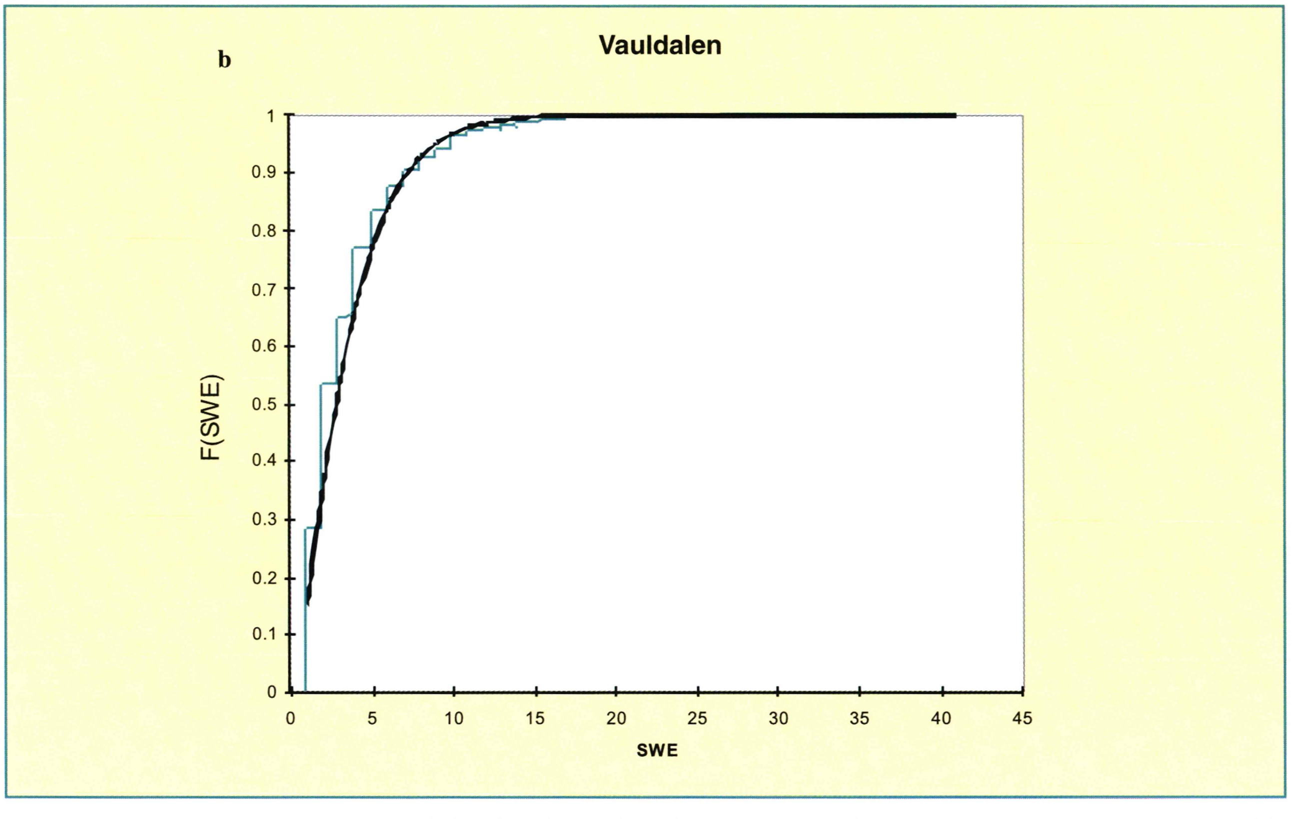
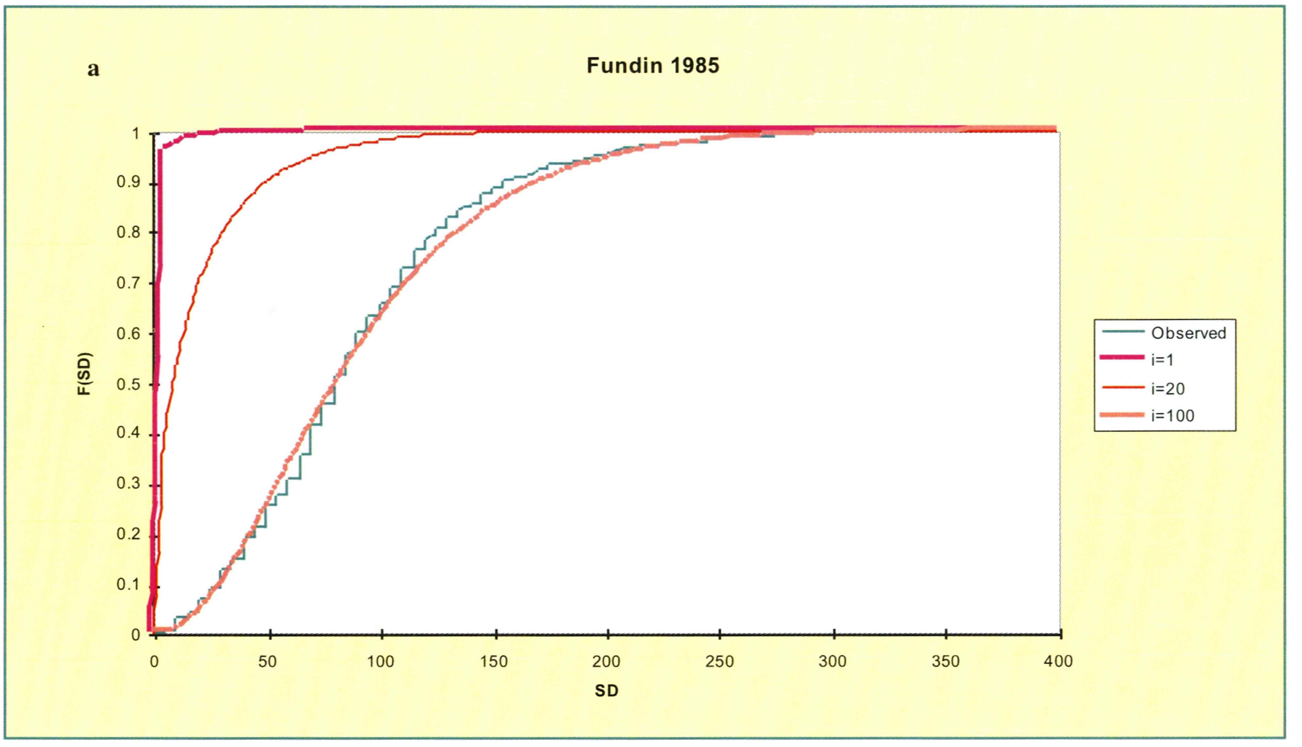


Figure 5. *Gamma distribution is fitted (thick solid line) to observed daily values for precipitation station 1040 (a) and snow pillow Vauldalen (b). Parameters for the gamma distribution are found in Table 1 (Vauldalen) and Table 2 (1040).*



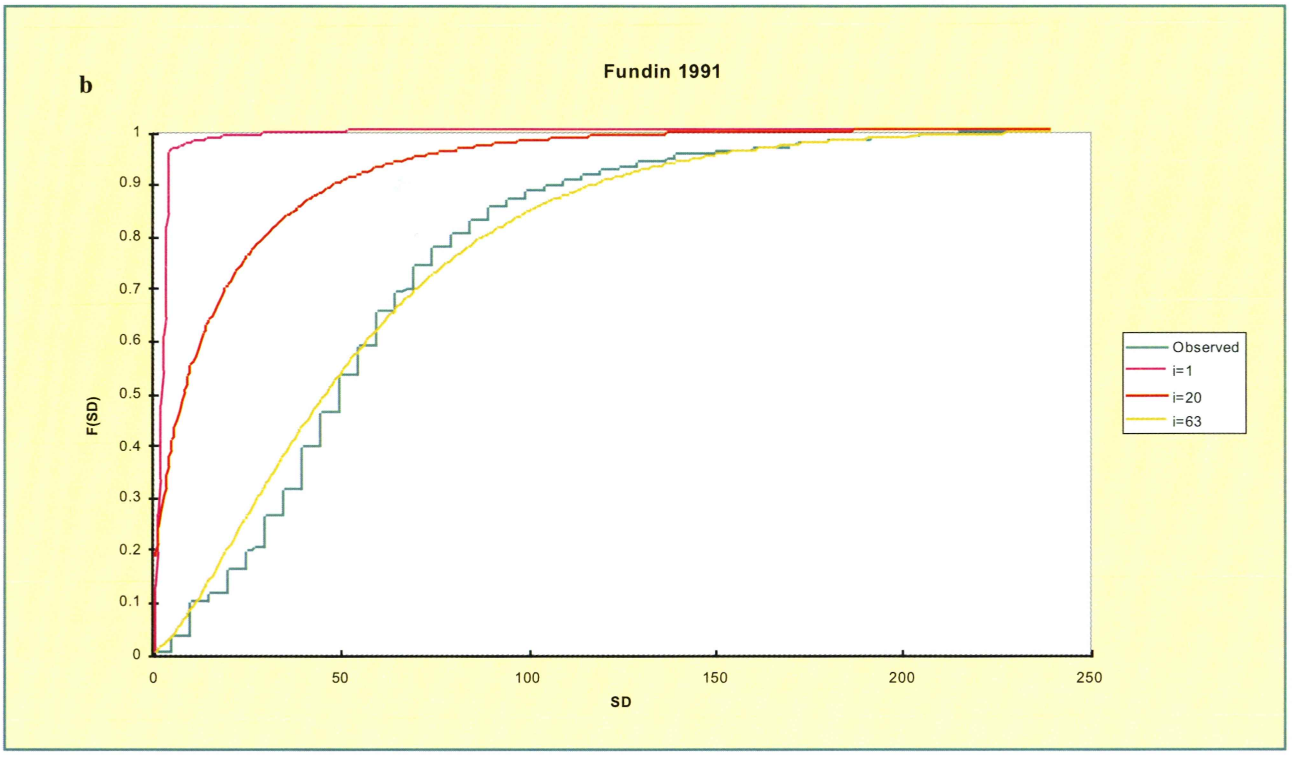


Figure 6. *Modelling possible development of snowpack by sums of gamma distributed variables with parametres* α*=0.027 and* ν*=0.0252, i= 1, i=20, i=100 (a), i=1, i=20, i=63 (b)*

Table 3. *Seasonal mean for snow pillows and precipitation gauges (only precipitation as snow and greater than 1 mm is in the computation). - - indicates insufficient data.*

	Seasonal mean of daily precipitation, E(SWE) [mm]						
Seasons (1/10-31/5)	Vauldalen	Filefjell	1040	871	813	1367	1674
1987/88	3.76	2.9	3.71	4.33	5.17	4.53	3.78
1988/89	4.01	5.12	4.22	4.29	3.72	5.36	4.70
1989/90	4.11	- -	3.67	3.97	6.25	4.61	3.89
1990/91	2.51	- -	3.74	3.68	4.3	3.98	2.60
1991/92	3.69	3.6	4.43	3.71	4.71	- -	4.32
1992/93	4.27	- -	3.61	3.77	5.79	4.51	3.14
1993/94	3.32	4.41	3.10	- -	4.35	4.02	3.84
1994/95	3.06	4.12	2.73	3.67	4.76	4.56	3.71
1995/96	2.66	2.44	3.47	- -	3.69	5.66	4.01
mean 1987/88-95/96	3.48	3.76	3.63	3.91	4.74	4.65	3.78

4.2 Properties of n

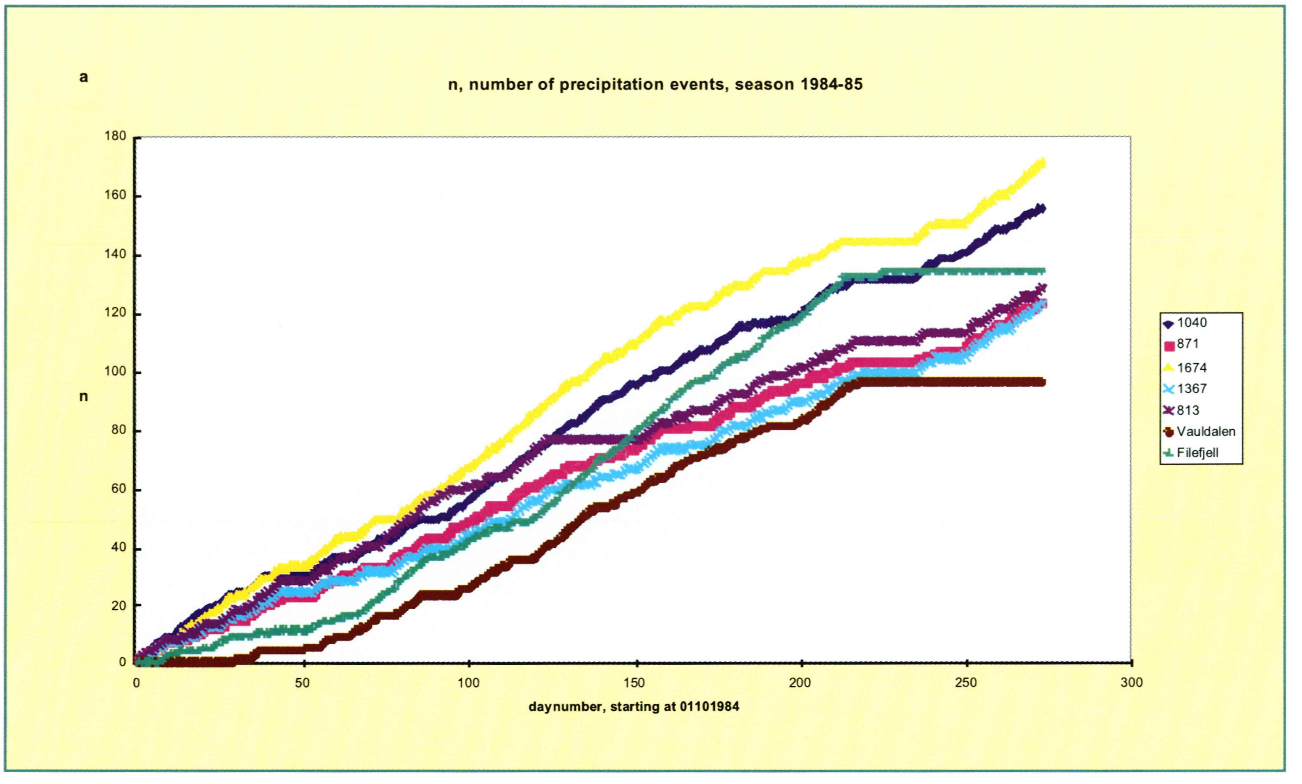
In the formulation of the estimator of the mean SWE over an area, we have assumed that n is the number of events of some spatial extension comparable to the area to be estimated. The assumption that n is a spatial property, can be checked by plotting n (the occurrence of a daily precipitation event, not necessarily snow fall, in that the snow pillow often registers precipitation also as a snowfall event) by day number for each season, which is defined (by the snow pillow data) to start from the start of October to the end of May/start of June, for the precipitation gauges and the snow pillows. Figure 7(a,b,c and d) shows four seasons (the winters -84/85, -86/87,-87/88 -94/95) for five precipitation gauges and two snow pillows. There is no systematic deviation of n depending on whether the source is a snow pillow or a precipitation gauge. We observe further that the two snow pillows agree quite well, although some discrepancy can be seen, often due to differences in the date when precipitation appeared as snow in the beginning of the season. The main conclusion that can be drawn concerning n, is that it appears as a variable describing the occurrence of a snowfall/precipitation event of some spatial extent.

4.3 Properties of a

According to (15), we are interested in the fraction of the area where snow is to be found, and a can be calculated in the standard fashion as the mean coverage:

$$a = E(sc) = \sum_{SC=0}^{100} sc \cdot p(sc), \qquad sc \geq 0 \tag{16}$$

where p is the discrete empirical probability density function for a particular snow coverage *(sc)*.



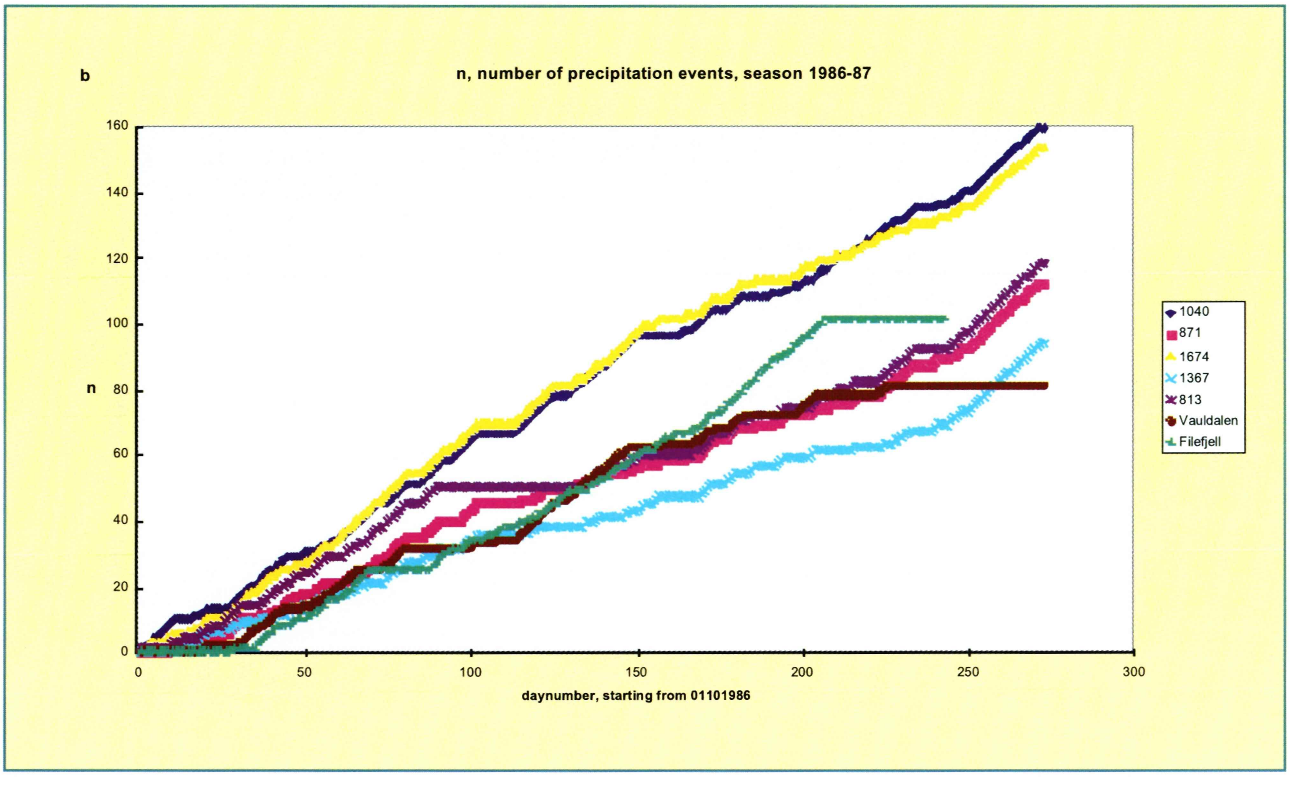
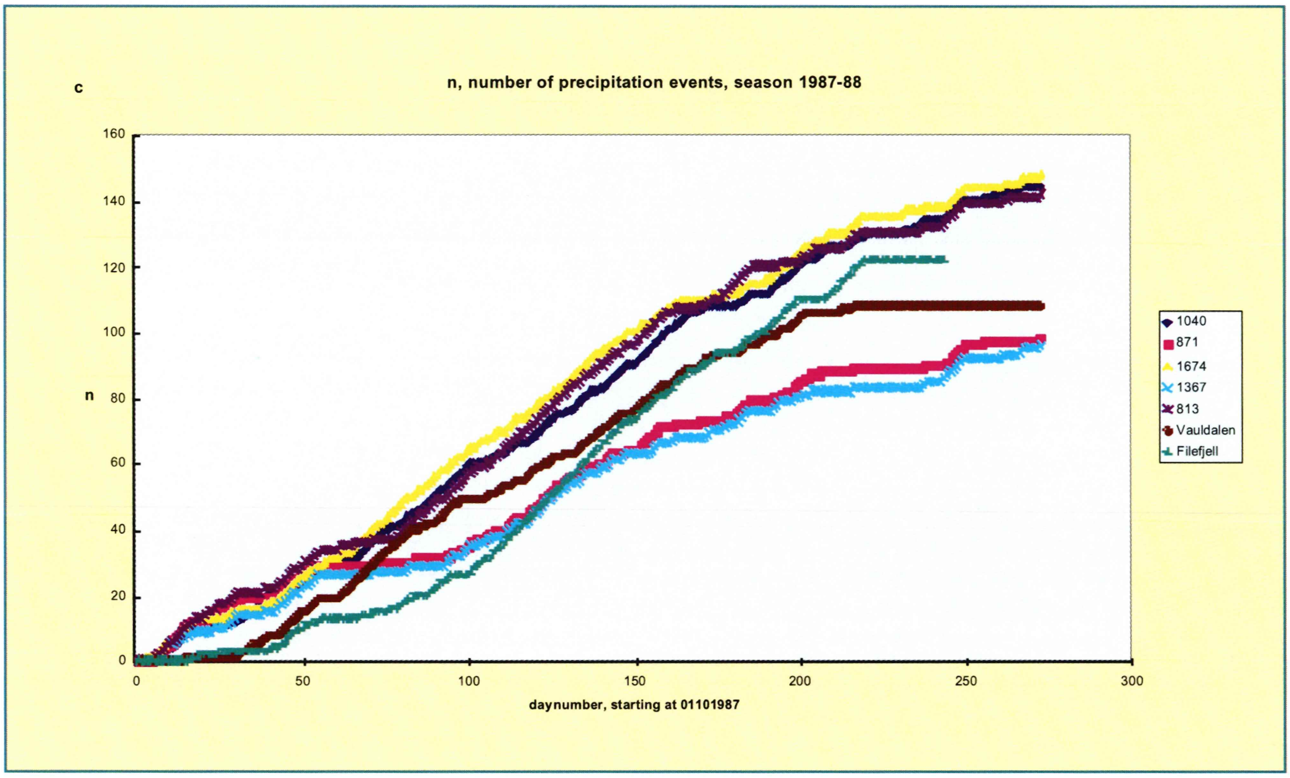


Figure 7. *Showing the number of precipitation events registered at precipitation stations, 1040, 871, 1674, 1367, 813 and at the snowpillows Vauldalen and Filefjell. a) Winter 1984-85, b) Winter 1986-87.*



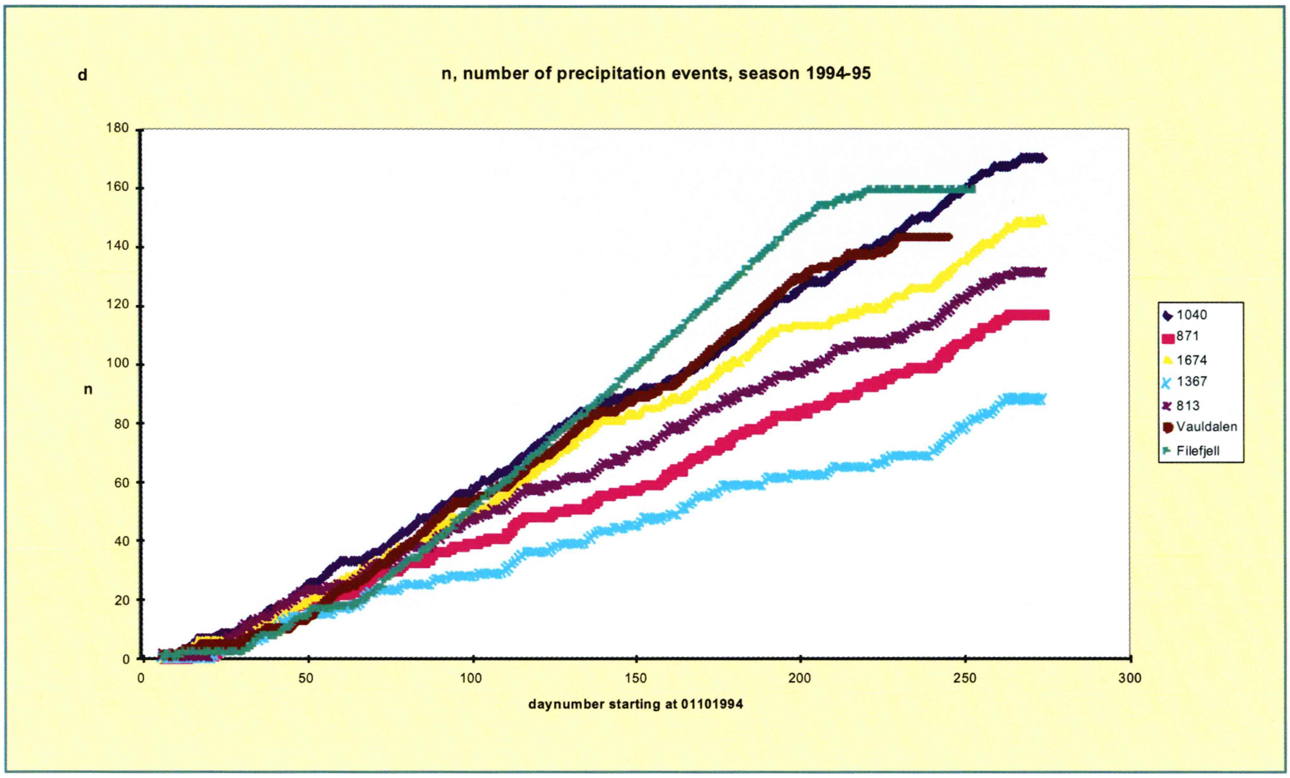


Figure 7. *(continued) Showing the number of precipitation events registered at precipitation stations, 1040, 871, 1674, 1367, 813 and at the snowpillows Vauldalen and Filefjell. c) Winter 1987-88, d) Winter 1994-95.*

5. Results and discussion

As is usual with the validation of areal estimates of hydro-meteorological variables, we are also in this case faced with the problem of an unknown truth. The use of snow courses is the usual way of estimating the volume of snow in advance of the melting season. For a validation procedure in this study, we need estimates of the mean SWE over specific areas at specific dates. The water management association GLB, has calibrated HBV models for all catchments in this basin. The HBV model (Bergstrøm, 1992) is a commonly used rainfall-runoff model in the Nordic countries, and a snowfall routine has been developed for operational use in Norway which accounts for the development of the snow pack and snow coverage at different altitude levels (Killingtveit and Sælthun, 1995). The estimates of mean SWE from the HBV-model are given for different catchments, which grouped, form the domains C1 and C2. The HBV estimates for C1 and C2 are weighted (by area) averages. Table 4. gives the parameters necessary for (15) together with the HBV-estimate and the estimate by (15) for the different dates. For C1, n and the parameter values for α and ν are from the Vauldalen snow pillow. For C2, n and the parameter values for α and ν are averaged for the snow pillows Vauldalen and Filefjell. For simplicity, a melting event is taken into account by simply reducing the numbers of accumulation events by one, even though Table 1 indicates that melting events are distributed with different parameter values than accumulation events.

A very general conclusion to the results of Table 4 is that the proposed method underestimates SWE compared to the HBV-model in late spring. We also note that there is a significant discrepancy between the snow coverage estimated by the HBV-model and by the satellite image (a). This is a systematic tendency for each year. From operational use of the HBV-model for flood forecasting/warning at the Norwegian Water Resources and Energy Administration, the model is known for not being able to simulate rapid melting. The consequence of this, is that at the end of the melting season, the HBV-model still has snow in the catchment, when satellite images and observed runoff signifies that no snow is left (pers. com. H. Hisdal, NVE). The discrepancy between the results of the proposed method and the HBV-model might be due to this effect. However, we must also take into account that estimating snow coverage from satellite images in late spring suffers from (at least) two conditions that systematically gives an underestimate. The exponential decrease in albedo as the number of days since the last snowfall (albedo=$0.85(0.82)^{\tau^{0.46}}$, where τ is the number of days since the last snowfall) (Bras, 1990, p.263), and areas covered with forest contain snow, which is undetected by the satellite. The selected areas for which we have made estimation of the mean SWE are generally situated in high altitudes, but forested areas are also present.

Table 4. *Validation of the proposed method with mean areal SWE estimates by the HBV-model.*

Date	Catchmente C1(4723 km^2) C2(19832 km^2)	n+	n-	a	α	ν	SWE (15) [mm]	SWE (HBV) [mm]	Snow coverage (HBV)	Δ SWE [mm]	$\frac{SWE_{HBV}}{SWE_{(15)}}$
290395	C1	110	8	0.77	0.46	1.62	275	265	1.0	10	0.96
	C2	121	12	0.77	0.41	1.50	305	413	1.0	-108	1.35
220595	C1	142	17	0.48	0.46	1.62	213	292	0.88	-79	1.37
	C2	152	23	0.56	0.41	1.50	263	403	0.8	-140	1.53
040695	C1	142	28	0.17	0.46	1.62	69	137	0.52	-67	1.98
	C2	152	35	0.32	0.41	1.50	135	216	0.47	-81	1.60
130695	C1	-	-	0.06	0.46	1.62	- -	78	0.34	- -	- -
	C2	162	48	0.19	0.41	1.50	79	149	0.33	-71	1.88
210396	C1	72	2	0.73	0.46	1.62	180	140	1.0	40	0.78
	C2	79	12	0.75	0.41	1.50	184	168	1.0	16	0.91
130496	C1	80	4	0.64	0.46	1.62	171	148	1.0	24	0.87
	C2	91	14	0.68	0.41	1.50	191	170	0.94	22	0.89
170397	C1	101	4	0.79	0.46	1.62	271	200	1.0	71	0.74
	C2	100	7	0.75	0.41	1.50	255	256	0.99	-1	1.0
040697	C1	128	19	0.22	0.46	1.62	84	161	0.52	-76	1.92
	C2	138	23	0.30	0.41	1.50	126	228	0.5	-102	1.85

The observed discrepancy in Table 4, besides the systematic underestimation of the proposed method in late spring can best be discussed if we can get a notion on how the HBV-model performs from year to year. HBV-estimated values have been compared with measured ones (snow course) on specific dates. The snow courses are done within 3-4 days, and areal values calculated from these measurements are compared with HBV-runs on close dates. Table 5 (a,b,c) shows the accuracy of the HBV-model. All catchments are contained within the test catchment C2.

We can get an indirect opinion on how the proposed method compares to the measured values by studying the ratio between values from the proposed method and the HBV-model, and the measured values and the HBV-model. The idea is to try to remove the possible bias in the HBV estimates. The ratios can be found in Table 4 and 5. Table 6 provides ratios for comparable dates.

Table 5a. *Comparison between HBV-estimated SWE and SWE estimated from snow courses for the season 1995.*

			1995			
Catchment	Area km^2	Measured [mm]	Date	SWE_{HBV} [mm]	Date (HBV)	$\frac{SWE_{HBV}}{Measured}$
Tesse	380	358	2104	442	2504	1.23
Breidal	137	726	1804	839	2504	1.16
Osen	1190	317	2503	273	2903	0.86
Aursund	830	302	2703	436	0104	1.44
Bygdin	308	825	0404	924	0104	1.12
Vinstra	162	586	2903	570	2903	0.97
Heimdalsvatn	128	484	2903	521	2903	1.08
Kaldfjord	104	423	2903	360	2903	0.85
Øyangen	42	300	2903	296	2903	0.99
Olstappen	636	272	2903	294	2903	1.08
Fundin/Elgsjø	245	324	0404	461	2903	1.42
Marsjø	23	233	0404	360	2903	1.55
Einunndal	220	252	0404	272	2903	1.08
Savalen	104	174	0404	196	2903	1.13
				Mean	$\frac{SWE_{HBV}}{Measured}$	1.21

Table 5b. *Comparison between HBV-estimated SWE and SWE estimated from snow courses for the season 1996.*

			1996			
Catchment	Area km^2	Measured [mm]	Date	SWE_{HBV} [mm]	Date (HBV)	$\frac{SWE_{HBV}}{Measured}$
Breidal	137	348	2603	350	2103	1.0
Aursund	830	212	1904	217	1904	1.02
Bygdin	308	278	1604	263	1604	0.95
Vinstra	162	187	1604	159	1604	0.85
Heimdalsvatn	128	157	1604	141	1604	0.90
Kaldfjord	104	137	1604	119	1604	0.87
Øyangen	42	99	1604	99	1604	1.0
Olstappen	636	97	1604	113	1604	1.16
Fundin/Elgsjø	245	185	3003	196	2103	1.06
Marsjø	23	122	3003	147	2103	1.20
Einunndal	220	149	3003	123	2103	0.83
Savalen	104	99	3003	113	2103	1.14
				Mean	$\frac{SWE_{HBV}}{Measured}$	1.0

Table 5c. *Comparison between HBV-estimated SWE and SWE estimated from snow courses for the season 1997.*

			1997			
Catchment	Area km^2	Measured [mm]	Date	SWE_{HBV} [mm]	Date (HBV)	$\frac{SWE_{HBV}}{Measured}$
Breidal	137	1053	0705	1145	1205	1.09
Bygdin	308	716	1604	692	1604	0.97
Vinstra	162	424	1604	375	1604	0.88
Heimdalsvatn	128	341	1604	380	1604	1.11
Kaldfjord	104	366	1604	244	1604	0.67
Øyangen	42	253	1604	179	1604	0.71
Olstappen	636	201	1604	128	1604	0.64
				Mean	$\frac{SWE_{HBV}}{Measured}$	0.87

Table 6. *The ratio between values from the proposed method and the HBV-model, and the measured values and the HBV-model for similar dates. Note that mean areal SWE is calculated for different catchments, so that the absolute values are not comparable.*

Dates	$\frac{SWE_{HBV}}{SWE_{(15)}}$	$\frac{SWE_{HBV}}{Measured}$
290395	0.96	0.97
	1.35	1.08
		0.85
		0.99
		1.08
		0.97
13-160496	0.87	0.95
	0.89	0.85
		0.9
		0.87
		1.0
		1.16

Based on the (very few) data at hand, there seems to be a better agreement between the proposed method and the snow courses, than both of them compared to the values from the HBV model. This is an indication that the estimates from the HBV-model should be treated with caution and that independent measures are needed.

A simplification has been made in treating melting events in the same manner as accumulation events (in the estimation procedure the number of accumulation events is the observed number of accumulation events minus the observed number of melting events). Table 1 shows that the mean and the parameter values for α and v for melting are different than for those of accumulation. The simplification is made because, unlike precipitation, the melting process can not be assumed homogenous in time, since melting, obviously is a function of incoming radiation, which increases regularly during spring. It would be complicated to treat the melting process as a non-homogenous gamma process, and this is left for further studies. However, the effect of simplifying with respect to the melting process should be that the snow volume is overestimated in late spring. It can be seen from Table 4 that the opposite is the case. Whether the underestimation is due to error in the other factors in the proposed method, or errors in the estimate made by the HBV-model, remains to be investigated.

It is interesting to note that the number of events where precipitation greater than 1 mm occurs at temperatures less than zero are in the neighbourhood of 1/3 of the number of events registered as accumulation on the snow pillows. This presents an important question to modellers of the snowmelt process. At what temperature is the precipitation accumulated?

One of the main inferences of this study is that, years of extreme amounts of snow does not necessarily imply that the snowfall events themselves are extreme, it can well be that the number of snow fall events is extreme. As mentioned in the introduction, the Glomma basin experienced an extreme flood due to melting and precipitation in June 1995. Several unfavourable factors occurred at the same time: 1) the spring was unusually cold. The melting started late and took place in several altitude levels simultaneously, 2) when the melting started, it was accompanied by heavy precipitation, and 3) the amounts of snow in the mountains was about 130-140% of the normal in late April. When we study the mean SWE for Vauldalen for the season 1994-95 (Table 3, $E(SWE_{1994\text{-}95})$=3.06), we find that it is less than the mean computed for all operational years ($E(SWE_{1987\text{-}95})$=3.48)). However, the number of accumulation events was large (see Table 4), and hence the extreme snow pack.

6. Conclusions

Different methods of estimating the mean areal SWE/snow depth from satellite images have been put forward. Of these, the modelling the snowfall process as sum of gamma distributed random variables has been found promising, in that it includes both spatial and point information.

The proposed method takes into account, that although the snow coverage, a, is similar, the number of precipitation events, n may vary, and give a different estimate to the mean SWE. It is the combination of n and a that provide the variability of the mean SWE over an area.

Based on very few data it seems that the proposed method performs similar to estimates of the mean areal SWE calculated from snow course measurements. The comparison is based on how the proposed method and snow courses perform compared to the HBV-model.

There is yet work to be done in the analysis of the satellite images to make the data from these suitable with sufficient accuracy to determine the coverage of the snow pack. The challenge is to improve the satellite data from being a supplementary tool to traditional, manual methods to monitor the snow cover to be able to analytically use the data.

Bearing this in mind, there should be a great potential for improving the method, both with increased density of snow pillow data and from improvement of the processing of the satellite images.

Acknowledgements

Hans Christian Udnæs of the National Water Resources and Energy Administration, has processed the satellite images and has prepared some of the figures. Dan Lundquist of GLB has provided data and thoughtful comments. Their help is gratefully acknowledged.

References

Bergstrøm, S., 1992. The HBV-model-its structure and its applications. SMHI RH No.4, Norrkøping, Sweden.

Bras, R., 1990, Hydrology, Addison-Wesley Pub. Comp. Inc., Reading, Mass., pp. 643.

Braud, I, J. D. Creutin and C. Barancourt, 1993. The relation between mean areal rainfall and the fractional area where it rains above a given threshold. J. Appl. Meteor., 32, 2, 193-202.

Faanes, T. and S. Kolberg, 1996. Optimal utnyttelse av snømagasinet. SINTEF report no. STF22 A96412 (in Norwegian).

Førland, E., P. Allerup, B. Dahlström, E. Elomaa, T. Jónsson, H. Madsen, J . Perälä, P Rissanen, H Vedin and F. Vejen, 1996. Manual for operational correction of Nordic precipitation data. DNMI report no. 24/96.

Feller, W. 1971, An introduction to probability theory and its applications. John Wiley, New York, pp. 669.

Gottschalk, L., 1995. Stokastisk hydrologi, Compendium, University of Oslo, Dept. of Geophysics, Oslo, Norway (in Swedish).

Gottschalk, L. and T. Jutman, 1979, Statistical analysis of snow survey data, Swedish meteorological and hydrological institute, SMHI, Report nr. 20.

Haan, C.T.,1977. Statistical methods in hydrology. The Iowa State University Press, Ames, Iowa, 378 pp.

Häggström, M., 1994. Snökartering i Svenska fjällområdet med NOAA-satellitbilder, Swedish meteorological and hydrological institute, SMHI, Report nr. 57 (in Swedish).

Kedem , B. L. S. Chiu and Z. Karni, 1990. An analysis of the threshold method for measuring area average rainfall. J. Appl. Meteor., 29 3-20.

Killingtveit, Å. and U. R. Sælthun, 1995. Hydrology, Volume no. 7, in Hydropower development. UiT, Trondheim, Norway.

Krzysztofowicz, R., 1997. Transformation and normalization of variates with specified distributions. Jour. Hydrol. 197, 286-292.

NOU 1996: 16. Tiltak mot flom. Nærings og energidepartementet, Oslo, Norway, NOU 1996: 16 (in norwegian).

Rango, A., 1996. Spaceborne remote sensing for snow hydrology applications., Hydr. Sci. Jour, 41(4), p. 477-494.

Schjødt-Osmo, O. R. Engeset, 1997. Remote sensing and snow monitoring: Application to flood forecasting, in Refsgaard, J.,C. and E. A. Karalis (eds.) Operational water management,. Proc. EWRA, Copenhagen-97. A. A. Balkema, Rotterdam.

Yevjevich, V., 1982, Probability and statistics in hydrology, Water resources publications, Littleton, pp. 302.

Kontaktpersoner

formann i styringsgruppen:
Erling Norvik
Tlf. 22 51 19 19

programleder:
Arnor Njøs
Jordforsk
Tlf. 64 94 81 70 (Jordforsk)
Tlf. 22 95 90 98 (NVE)
E-post: arnor.njos@jordforsk.nlh.no
E-post: xarn@nve.no.

naturgrunnlag og arealbruk:
Arne Grønlund
Jordforsk
Tlf. 64 94 81 09
E-post: arne.gronlund@jordforsk.nlh.no
Noralf Rye
Universitetet i Bergen
Tlf. 55 58 34 98
E-post: noralf.rye@geol.uib.no

tettsteder:
Oddvar Lindholm
Norges Landbrukshøgskole
Tlf. 64 94 87 08
E-post: oddvar.lindholm@itf.nlh.no

flomdemping, flomvern og flomhandtering:
Dan Lundquist
Glommens og Laagens
Brukseierforening
Tlf. 22 54 96 00
E-post: post@glb.no
E-post: danlund@sn.no

skaderisikoanalyse:
Nils Roar Sælthun
Norsk institutt for vannforskning
Tlf. 22 18 51 21
E-post: nils.saelthun@niva.no

miljøvirkninger av flom og flomforebyggende tiltak:
Olianne Eikenæs
Norges vassdrags- og energidirektorat
Tlf. 22 95 92 24
E-post: oli@nve.no

databaser og GIS:
Lars Andreas Roald
Norges vassdrags- og energidirektorat
Tlf. 22 95 92 40
E-post: lars.roald@nve.no

modellarbeid:
Ånund Killingtveit
Norges teknisk-naturvitenskaplige universitet
Tlf. 73 59 47 47
E-post: aanund.killingtveit@bygg.ntnu.no

programadministrasjon:
Olianne Eikenæs
Norges vassdrags- og energidirektorat
Tlf. 22 95 92 24
E-post: oli@nve.no
Hjemmeside: http://www.nve.no
Per Einar Faugli
Norges vassdrags- og energidirektorat
Tlf. 22 95 90 85
E-post: pef@nve.no

Norges
vassdrags- og
energidirektorat

NVE

Kontoradresse: Middelthuns gt. 29
Postadresse: Postboks 5091 Maj. 0301 Oslo

ISBN 82-410-0347-1