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Simulated precipitation fields with variance-consistent interpolation

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Abstract Gridded meteorological data are available for all of Norway as time series dating from 1961. A new way of interpolating precipitation in space from observed values is proposed. Based on the criteria that interpolated precipitation fields in space should be consistent with observed spatial statistics, such as spatial mean, variance and intermittency, spatial fields of precipitation are simulated from a gamma distribution with parameters determined from observed data, adjusted for intermittency. The simulated data are distributed in space, using the spatial pattern derived from kriging. The proposed method is compared to indicator kriging and to the current methodology used for producing gridded precipitation data. Cross-validation gave similar results for the three methods with respect to RMSE, temporal mean and standard deviation, whereas a comparison on estimated spatial variance showed that the new method has a near perfect agreement with observations. Indicator kriging underestimated the spatial variance by 60–80% and the current method produced a significant scatter in its estimates.

Key words spatial rainfall; interpolation; spatial variance; intermittency; Norway

Simulation de champs de précipitation par interpolation cohérente en termes de variance
Résumé Des séries temporelles de données météorologiques maillées sont disponibles pour l'ensemble de la Norvège depuis 1961. Une nouvelle façon d'interpoler les champs de précipitations dans l'espace à partir des valeurs observées est proposée. Sur la base des critères selon lesquels les champs de précipitations interpolés dans l'espace devraient être compatibles avec les statistiques spatiales observées comme les moyennes, variances et intermittences spatiales, les champs de précipitations sont simulés selon une distribution gamma déterminée à partir de données observées, ajustées pour l'intermittence. Les données simulées sont distribuées dans l'espace à l'aide du patron spatial dérivé par krigeage. La méthode proposée est comparée à l'indicateur de krigeage et à la méthode actuellement utilisée pour produire des données de précipitations maillées. La validation croisée a donné des résultats similaires pour les trois méthodes, pour les valeurs de l'erreur quadratique moyenne, de la moyenne temporelle et de l'écart type, tandis que la comparaison sur la variance spatiale a montré que la nouvelle méthode donne un accord presque parfait avec les observations. L'indicateur de krigeage sous-estime la variance spatiale de 60-80% et la méthode actuelle produit une dispersion significative de ses estimations.

Mots clefs champs de précipitations; interpolation; variance spatiale; intermittence, Norvége

INTRODUCTION

Spatial interpolation of meteorological elements such as precipitation and temperature is essential in operational hydrology. In Norway, the national flood forecasting service is based on simulations and forecasts from 117 catchments, using the Swedish hydrological model, the HBV model (Bergström, 1992). In order to facilitate rapid inclusion of new catchments into the forecasting system, it has been decided to use meteorological grids of temperature and precipitation as input to the hydrological models rather than weighted values from nearby specific meteorological gauging stations. The problem with using specific gauging stations is that these stations have a varying life span dependent on their representativeness, and the availability of staff to carry out the observations. Once such a station is no longer operational, the hydrological forecast model has to be re-calibrated for a new set of gauging stations. It is foreseen that maintenance of the flood forecasting system would be much easier if meteorological grids were used. However, these grids have to be interpolated on a daily basis according to available gauging stations. The current precipitation grid (see examples at www.senorge.no) is obtained by a simple interpolation method, where a triangular

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irregular network (TIN) of precipitation is established based on all points of observation. A grid of resolution 1×1 km^2 is constructed from the TIN. A similar grid of station altitude is produced and this altitude grid is compared to altitudes from a digital terrain model (DTM); then the precipitation grid is adjusted using a precipitation gradient of 10%/100 m on the differences between the two altitude grids (Jansson *et al.*, 2007). The resulting precipitation grid compared well with grids obtained by other methods used in the Nordic countries (Jansson *et al.*, 2007). However, the method has the disadvantage of not being able to extrapolate outside defined triangles, and the spatial variability and correlation are, for a large part, the result of topography rather than derived from the rainfall process itself.

Most methods used for interpolating precipitation, even sophisticated methods such as kriging (Creutin & Obled, 1982; Teegavarapu, 2007), have a tendency to produce too smooth fields, i.e. to underestimate the spatial variability (Creutin & Obled, 1982; Haberland, 2007). The loss of variability will affect the estimation of extreme values and undermine the main contribution of the distributed hydrological model which is the optimal use of spatially distributed moisture input. The relative importance of the spatial variability of rainfall in relation to hydrological prediction is not entirely clear, and is dependent on physical catchment characteristics, catchment scale, precipitation type and scale, antecedent moisture conditions and type of hydrological model (lumped, distributed, physical, conceptual, etc.; Segond *et al.*, 2007). Most authors conclude, however, that for semi-distributed and distributed models, improvements in predictions and hydrological insights are gained by taking into account the spatial variability of rainfall (Singh, 1997; Tetzlaff & Uhlenbrook, 2005; Pechlivanidis *et al.*, 2008). Chaubey *et al*. (1999) point out that a large uncertainty in estimated parameters of the hydrological model can be expected if spatial variability of precipitation is ignored.

Ideally, we would like an interpolated rainfall field to have the same spatial statistical features as the observed spatial fields. The spatial statistical features that are the focus of this study are spatial mean, spatial variability and realistic fractions of dry area. The path followed in this study is to simulate precipitation values according to a two-parameter gamma distribution. The parameters are estimated from within-storm spatial mean and variance. By the term “within-storm”, we mean the part of the area in question in which positive values of precipitation are observed (see Seo, 1998). The within-storm spatial mean and variance are derived from the observed moments, but with estimated intermittency taken into account. Simulated precipitation values typically span outside the interval determined by the smallest and largest observed values (apart from observed zeros) and we try here to devise rules to correctly place the maximum simulated precipitation value in space. We further hypothesize that this added information will result in more precise interpolated fields. Furthermore, we use the spatial distribution of a kriged field (using spatially mapped ranks of precipitation values) as a map of where to distribute the simulated values. The new method for simulating/interpolating precipitation fields is hereafter named SPF, an abbreviation for its product, simulated precipitation fields.

Over a catchment, precipitation is often fractional, and procedures for the interpolation of precipitation thus have to take into account within-storm variability and the variability due to intermittency (Seo, 1998). Here, the intermittency is estimated through the relationship between observed spatial mean and variance and derived within-storm spatial mean and variance. The proposed method for determining intermittency is compared to that of indicator kriging (Barancourt *et al.*, 1992) and to the method of deriving intermittency from weather radar. Indicator kriging is also the method to which SPF is compared through cross-validation analysis.

In the next section, we present the theory applied for estimating intermittency and thus the within-storm parameters in the gamma distribution. The methods derived for mapping the simulated precipitation values are also described in this section. The next section presents the results of the cross-validation and a discussion follows in the last section.

METHOD

The methods developed in this paper are described below: first the statistical model, the gamma distribution, used for simulating precipitation is presented; then, we introduce an algorithm for estimating intermittency from observed spatial mean and variance. Next we develop a method for determining a possible location for the highest simulated value, and, finally, the method for distributing the simulated precipitation in space is presented.

Statistical model

Let us introduce a statistical model for the spatial distribution of within-storm positive precipitation, $z'(x)$, ($z'(x) > 0$). Precipitation $z'(x)$ is measured during a fixed time interval (in this case daily) across space, x. In order to further simplify the notation, we omit x, so positive rainfall across space is hereafter denoted z'. The distribution in space of the within-storm (positive) rainfall is assumed to follow a gamma marginal distribution.

$$f_{\alpha,\nu}(z') = \frac{1}{\Gamma(\nu)} \alpha^{\nu} z'^{\nu-1} e^{-\alpha z'} \quad \alpha, \nu, z' > 0 \tag{1}$$

where ν and α are the shape and scale parameters, respectively. The mean is $E(z') = \nu/\alpha$ and the variance $\text{var}(z') = \nu/\alpha^2$. The parameters are estimated from the observed spatial mean and variance as:

$$\alpha = \frac{\hat{E}(z')}{\hat{\text{var}}(z')} \qquad \nu = \frac{\hat{E}(z')^2}{\hat{\text{var}}(z')} \tag{2}$$

where the hats denote the estimate of the expected value.

The gamma distribution is a common choice of distribution for precipitation in space due to its flexibility in shape (Robinson & Sivapalan, 1997; Kuzuha *et al.*, 2006), which may range from very skewed to the left or approaching that of the normal distribution.

When spatial precipitation is studied over a fixed area, we find two possible outcomes of the spatial pattern: (a) the area might be fully covered with precipitation; and (b) we have an intermittent field, where only a fraction of the area is covered by precipitation. For both cases, the spatial distribution of precipitation can be described by a two-parameter gamma distribution as suggested above.

Estimating the intermittency of a precipitation field

In the case of full coverage of precipitation over the area of interest, fitting the gamma distribution to the observed spatial mean and variance is straightforward, and equation (2) is used to estimate the parameters. In the case of intermittency, we are faced with the problem of having to determine the fraction of wet area p, within a fixed domain in order to decide on the parameters of two-parameter gamma distribution for the within-storm precipitation. Let z and z' denote precipitation distributed in space including and not including zeros, respectively. We refer to the within-storm precipitation, z', as conditional precipitation (conditioned on positive precipitation) and to z, as non-conditional precipitation. In order to determine the fraction of wet area, p, we have to express p as a function of conditional and non-conditional moments. Furthermore, if we make an assumption on the functional relationship between the non-conditional spatial mean and variance, we have enough information to determine the fraction of wet area p and the parameters of the within-storm distribution of precipitation.

If we grid a fixed area into n elements, and m of these elements are wet, the first-order moments of non-conditional and conditional precipitation are, respectively:

$$\begin{aligned} E(z) &= \frac{n-m}{n} 0 + \frac{m}{n} E(z') \\ &= pE(z') \end{aligned} \tag{3}$$

$$E(z') = \frac{E(z)}{p} \tag{4}$$

where $p = m/n$ is the fraction of the area with positive precipitation. Similarly, for the second-order moments:

$$\begin{aligned} E(z^2) &= \frac{n-m}{n} 0 + \frac{m}{n} E(z'^2) \\ &= pE(z'^2) \end{aligned} \tag{5}$$

$$E(z'^2) = \frac{E(z^2)}{p} \tag{6}$$

From equation (6), one can write the conditional variance, $\text{var}(z')$, as

$$\begin{aligned} \text{var}(z') &= E(z'^2) - E(z')^2 \\ &= \frac{E(z^2)}{p} - E(z')^2 \end{aligned} \tag{7}$$

and since $E(z^2) = \text{var}(z) + E(z)^2$ equation (7) can be written as:

$$\text{var}(z') = \frac{\text{var}(z) + E(z)^2}{p} - E(z')^2 \tag{8}$$

which gives:

$$p = \frac{\mathrm{var}(z) + \mathrm{E}(z)^2}{\mathrm{var}(z') + \mathrm{E}(z')^2} \tag{9}$$

From equation (3), we also have:

$$p = \frac{\mathrm{E}(z)}{\mathrm{E}(z')} \tag{10}$$

which, together with equation (9), give:

$$\frac{\mathrm{var}(z) + \mathrm{E}(z)^2}{\mathrm{E}(z)} = \frac{\mathrm{var}(z') + \mathrm{E}(z')^2}{\mathrm{E}(z')} \tag{11}$$

From observed values we can determine the non-conditional mean and variance, $\hat{\mathrm{E}}(z)$and $\hat{\mathrm{v}}\mathrm{ar}(z)$, and the right-hand side of equation (11) is quantified. In order to proceed further and quantify the fraction of wet area, p, and the conditional mean and variance, $\mathrm{E}(z')$ and $\mathrm{var}(z')$, we introduce an assumption regarding the regularity of the relationship between $\mathrm{E}(z')$ and $\mathrm{var}(z')$. If we assume a one-to-one functional relationship between $\mathrm{E}(z')$ and $\mathrm{var}(z')$, equation (11) becomes an equation with only one unknown. With estimated values of $\mathrm{E}(z')$ and $\mathrm{var}(z')$, the fraction of wet area, p, can be determined from equation (10). In support of an assumption of a one-to-one relationship between $\mathrm{E}(z')$ and $\mathrm{var}(z')$, we have found reports in the literature on very high correlation between spatial mean and spatial standard deviation (Creutin & Obled, 1982; Barancourt *et al.*, 1992; Skaugen, 2007), which we can use to determine the right-hand side of equation (11). In assessing this relationship, we have to consider events, over a fixed area, for which zeros are observed, but where spatial mean and variance are estimated from non-zero observations. This will ensure that the observed spatial distribution is not bounded to the left by positive values. Of course, the data available for this assessment are limited, as there are not so many events (especially events with heavy precipitation) for which there are also stations within the same area that measure zero precipitation. Figure 1 shows an assessment of the fixed relationship between standard deviation and mean for conditional precipitation of the Rissa area in central Norway.

If $\mathrm{var}(z')$, or rather the standard deviation, $s(z')$, is considered as a function of $\mathrm{E}(z')$, the left-hand side of equation (11) can be compared, for different values of $\mathrm{E}(z')$, with the right-hand side of equation (11), and the wet fraction can be estimated through equation (10), when a match is found.

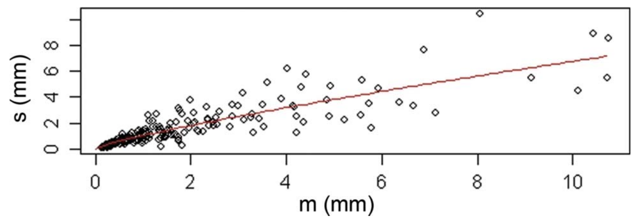

Fig. 1 Fixed relationship between spatial standard deviation and mean (conditional values) for the Rissa area. The fixed relationship is modelled as $s = am^h$, where s and m are estimated standard deviation and mean respectively and a (1.012) and h (0.83) are parameters.

It should be noted here that the reason for estimating the intermittency through the proposed method instead of using the fraction of observation points that measure zero precipitation is that the number of precipitation gauges in the area of interest is often too small to give us a reliable estimate of the fraction of wet area.

Location of the highest simulated value in space

This section describes the non-trivial task of placing the highest simulated precipitation value in space. When interpolating using kriging, the highest value in the kriged field is equal to, or very similar, to the highest observed. However, the highest simulated value is almost always higher than the highest observed, and we need to place this value at a location which is consistent with the pattern of the observed values. This is necessary for the procedure described in the next sub-section, which concerns the distribution in space of all the simulated values, and in which the highest simulated value is treated as an observation. In order to avoid confusion, we wish to make it clear that kriging interpolation is carried out twice in SPF. First we use the kriged field to place the highest simulated value in space, according to the procedure described herein. Second, we apply kriging again, using the highest simulated value as an observation, and use the kriged field when distributing all the simulated values in space. The latter procedure is described in the next subsection.

It is difficult to devise clear rules as to where a precipitation maximum should be located. An intuitive thought is to associate maximum precipitation with topography, i.e. placing the maximum value at the highest peak close to the highest observed values. Such a procedure was abandoned based on reported low correlation between precipitation and altitude in

Norway. Two Norwegian studies conclude that the altitude explains very little of the variability for daily, monthly and annual precipitation, and negative trends are often found (Førland, 1979; Steinsland *et al*., 2007). Furthermore, when such a procedure was tested using cross-validation, we found that for some locations estimation was improved compared to kriging, whereas for other locations this was not the case. Interestingly enough, placing the highest precipitation value at the lowest local location was found to be just as good. The adopted procedure is a result of a trial-and-error process, in which several schemes were tested and evaluated; it can be described as follows: the optimum location of the highest simulated value is the location where the spatial correlation structure in terms of a semivariogram is best defined. For precipitation stations rather close to each other, we expect a close-to-linear slope in a well-defined empirical semivariogram (see Cole & Moore, 2008). The winning location is thus selected as the point for which the deviations from a linear slope of the empirical semivariogram are the smallest. In practice, the highest simulated value is placed in different locations and the empirical semivariogram is calculated for each new location. The location which gives the smallest deviations from the linear slope of the empirical semivariogram is defined as the optimum location of the highest simulated value. However, it can be quite time-consuming to test every possible location within an area of interest, so the area for testing locations is restricted in the following way. The precipitation field is interpolated by ordinary kriging from the observed values and ranked. It is reasonable to assume that the highest simulated value should be located in a general area where we find the highest kriged values. The size of the area, in which we assume the highest simulated value is located, is restricted to the number of simulated values that are higher than the highest observed value. Note that each value represents the average value within a grid cell so that a collection of values also represents an area. This procedure defines an area which is large when the difference is large between the highest simulated and observed value (i.e. we have a field with high spatial variability), and small when the difference is small.

The procedure described above gave the best result when evaluated using cross-validation. Other schemes, based, for example, on precipitation gradients and correlation distances, were inferior to the chosen method, but the problem should remain open for further study.

Distribution of simulated precipitation in space

As already stated, the main objective of this study is to produce daily precipitation fields which respect the observed spatial mean, variance and intermittency. In order to achieve this we choose to simulate values from a gamma distribution with parameters estimated so that simulated spatial mean and variance are the same as the observed values. The following description deals with the method of distributing the simulated values in space with the aid of the kriging interpolation method.

In distributing the simulated values in space, we use the same line of reasoning as in Skaugen (2002). A precipitation field is interpolated from the observed values using ordinary kriging. Here, the highest simulated value is included as one of those observed through the procedure described in the previous sub section. The vector of *kriged* values is ranked and the spatial location is determined from the location in the vector (i.e. the first value corresponds to the lower left value in the grid). So to each spatial location, there is assigned a rank, denoting the relative precipitation value (the highest, the second highest, etc.). The vector of *simulated* values is sorted and each simulated value is placed at the location where its place in the simulated vector (after ordering) and rank from the kriged field are equal. In mathematical terms this procedure can be described as follows (see also Skaugen, 2002): let $\boldsymbol{V}$ be a vector of ordered simulated values, drawn from a gamma distribution and sorted into ascending order. Let $\boldsymbol{Y}$ be the vector of kriged values. Then if **rankY** is a vector of the ranks of the kriged values $\boldsymbol{Y}$, the resulting precipitation field $\boldsymbol{Z}$, can be expressed as:

$$\boldsymbol{Z}[i] = \boldsymbol{V}[\mathbf{rankY}[i]] \quad \text{for all } i = 1, \ldots, I \tag{12}$$

where the brackets [ ] indicate the components of the vector and also the spatial location. The result is thus a field with gamma distributed values of non-zero precipitation, and with spatial dependencies inherited from the interpolated field Y.

Figure 2 shows a simplified flow chart of the steps and procedures necessary for producing simulated precipitation fields.

RESULTS

In this section we will first evaluate the algorithm for estimating the intermittency. Finding a sufficiently dense network of precipitation gauges in Norway to estimate the intermittency is difficult. A possible way

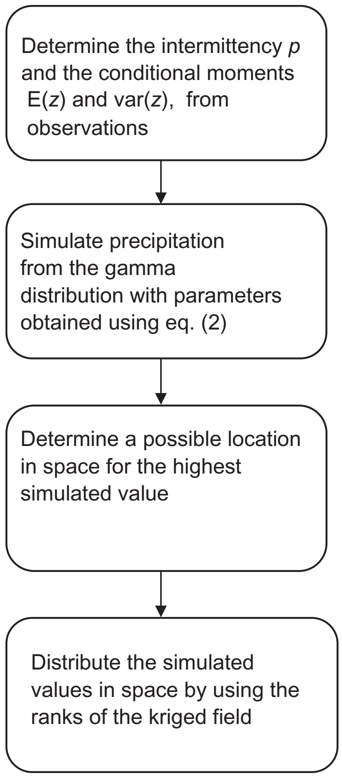

Fig. 2 Simplified flow chart of SPF procedures.

is to use observations from weather radar. The Rissa radar in central Norway (near Trondheim) provided precipitation observations (derived by the Norwegian Meteorological Institute) on a 1 × 1 km^2 grid for the Øyungen catchment (238 km^2, with elevations 103–680 m a.s.l.). Figure 3 shows the location of the radar and some nearby catchments.

The Øyungen catchment is located quite near the radar and the radar is considered to provide good precipitation estimates. Through the observed non-conditional spatial mean and variance of precipitation (derived by the radar) and the fixed relations between conditional moments (see Fig. 1), we can estimate the intermittency. Figure 4 shows how the estimated intermittency compares with intermittency observed by the radar. The agreement is good, especially for precipitation events of small spatial extent.

Cross-validation comparison between SPF and indicator kriging (IK)

In order to evaluate SPF, cross-validation was performed for two areas in Norway: Oslo (interpolating over a grid of 60 × 60 km^2) and Trondheim (interpolating over a grid of 70 × 70 km^2). Cross-validation is a procedure in which values of one of the observed stations are removed when interpolating. The values of this particular station are then estimated by the interpolation method and compared to what is observed. Cross-validation was carried out for nine stations in the Oslo area (ranging in altitude from 53–514 m a.s.l.) and 10 stations in the Trondheim area (11–158 m a.s.l.) using a time series of length

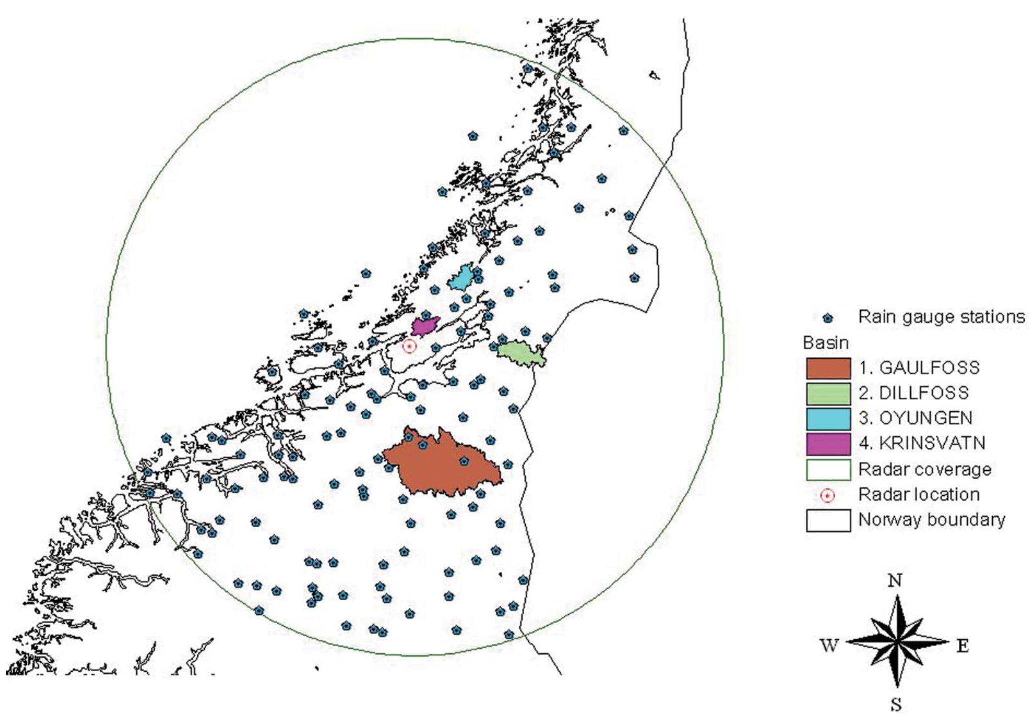

Fig. 3 Location of weather radar and the Øyungen catchment in central Norway.

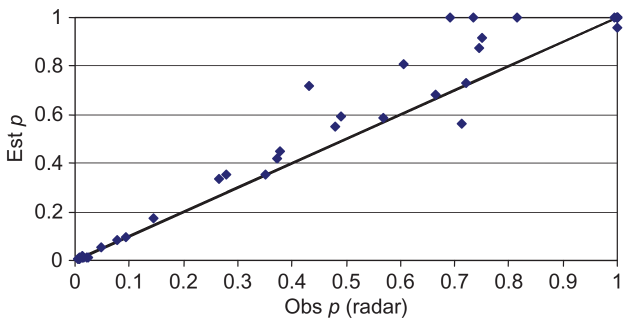

Fig. 4 Estimated *vs* observed intermittence (329 events, radar data for Øyungen), where p is the fraction of wet area.

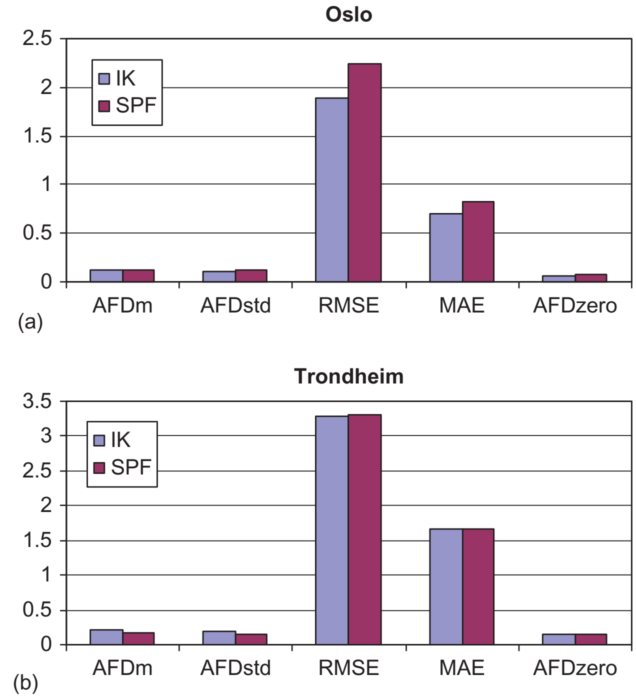

Fig. 5 Assessing the performance of SPF and IK against observed values for (a) the Oslo area; and (b) the Trondheim area. The best method evaluated by the different measures is the one with the shortest bar.

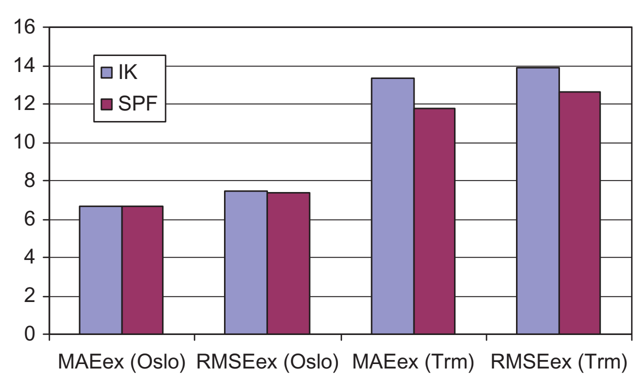

Fig. 6 Assessing the performance of SPF and IK for extreme values at Oslo (bars to the left) and Trondheim (bars to the right). The best method evaluated by the different measures is the one with the shortest bar.

one year. We wanted to compare the time series obtained by SPF and by indicator kriging (IK) with the observed time series in order to evaluate the relative performance of SPF compared to IK. Kriging is a very commonly-used interpolation method (Teegavarapu, 2007) and is also considered to be among the best interpolation methods for precipitation (Creutin & Obled, 1982; Lebel, 1987). In indicator kriging, the probability of precipitation is mapped spatially. If precipitation is observed, then the station in question is assigned the value one, and zero if not. A probability field is then interpolated. If a pixel in space has the probability of more than 0.5 for precipitation, the value obtained from ordinary kriging is used and a value of zero precipitation is assigned if the probability of precipitation is below 0.5. Ordinary kriging was carried out using an exponential semivariogram model with sill equal to the observed spatial variance and with a nugget effect equal to 5% of the sill. The results of the cross-validation for SPF and IK are shown in Fig. 5. The measures used for evaluation are: mean absolute error (MAE), the root mean square error (RMSE), absolute fractional deviation AFD, (AFD = |1 – (sim/obs)|, where sim is the simulated value and obs the observed value) of temporal mean (AFDm), standard deviation (AFDstd), and number of dry days (AFDzero).

In general, the performance of the two methods appears to be very similar. The IK method is somewhat better for the Oslo area, and the two methods perform very similarly for the Trondheim area. Note that the amount of precipitation is generally much higher for the Trondheim area with a daily mean for this year of about 3–4 mm whereas for the Oslo area the daily mean was less than 2 mm.

Extreme values

In order to investigate the ability of SPF and IK to estimate extreme values, the two highest observed values during the year of observation were compared to those estimated. Figure 6 shows the MAE and RMSE for the extreme values. The extreme values for the Trondheim area were best estimated by SPF, whereas the two methods have the same performance for the Oslo area. We also compared the sample quantiles obtained for the cross-validation series of SPF and IK for the probabilities 0.8, 0.82, up to 1.00 against the sample quantiles of the observed precipitation. A mean AFD of the quantiles, averaged over the different probabilities was computed for each station and this value was averaged for all the stations within the areas Oslo and Trondheim. The SPF performed marginally better for the Oslo area with a mean AFD over the quantiles and over the stations with 0.14 (SPF) against 0.15 (IK), and clearly better for the Trondheim area with 0.19 (SPF) against 0.23 (IK).

Comparison of SPF and the current interpolation method

The SPF method was also compared to the current interpolation method for precipitation, used in www.senorge.no (denoted SeNorge). The method is briefly described in the introduction. Figure 7 shows the results of comparing SPF and SeNorge derived values against independent observations of precipitation for the month of August 2001. The independent precipitation values are independent in the sense that they were not used in estimating the SPF or SeNorge precipitation fields.

We observe that SPF is just slightly better than the current interpolation method. As, in the case of the cross validation exercises, SPF compares well with the mean and standard deviation of the observed time series, whereas for the absolute and squared deviations (MAE and RMSE), no improvements of SPF compared to the other methods are found.

Comparison of spatial fields

We also compare the spatial statistics of the estimates of SPF, IK and SeNorge. The estimated spatial means and variances were compared to those observed from the network of precipitation gauges for the two areas. The results are shown in Figs 8 and 9.

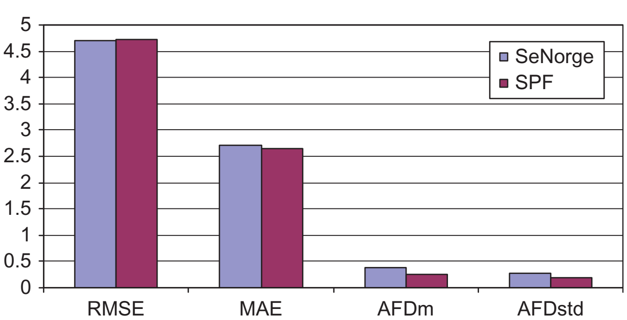

Fig. 7 Assessing the performance of SPF and the current interpolation method used at the SeNorge area. The best method evaluated by the different measures is the one with the shortest bar.

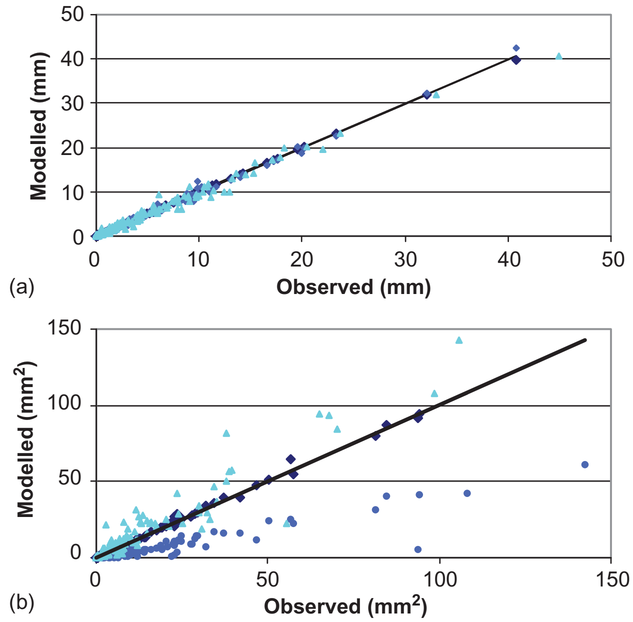

Fig. 8 (a) Modelled and observed spatial mean and (b) variance for the Oslo area. Small diamonds denote IK, large diamonds – SPF, and triangles – SeNorge.

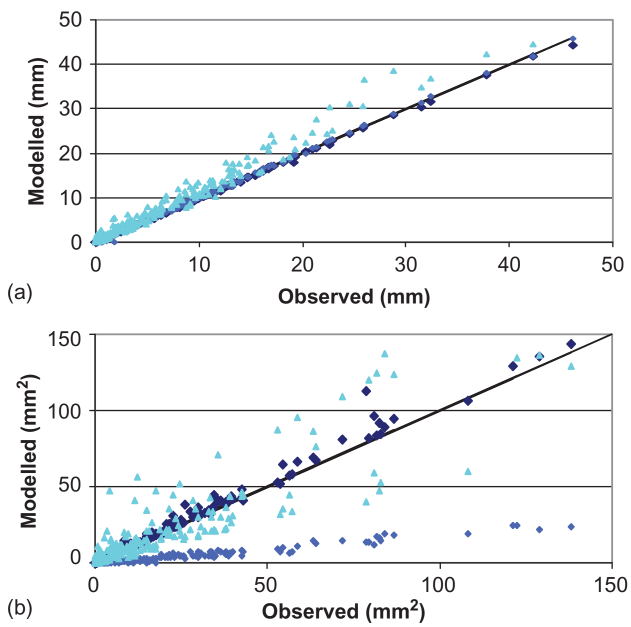

Fig. 9 (a) Modelled and observed spatial mean and (b) variance for the Trondheim area. Small diamonds denote IK, large diamonds – SPF, and triangles – SeNorge.

The comparison shows that, whereas the estimates of the spatial mean are indistinguishable for IK and SPF, SeNorge appears to have a very small negative bias for the Oslo area and a clear positive bias for the Trondheim area. In regard to the estimation of spatial variance, the most striking result is the severe underestimation of spatial variance by IK. The kriged fields reproduce 40% of the observed spatial variability for the Oslo area and only 20% for the Trondheim area. The SPF method naturally estimates the spatial variance very well, as the simulations are performed with a specified variance equal to that of the observed precipitation values, whereas SeNorge exhibits an overestimation for the Oslo area and a slight underestimation for the events of low intensity for the Trondheim area. A considerably larger scatter is observed for SeNorge compared to SPF.

The estimates of the fraction of wet area (p) also deserve a comment. Figure 10 shows the estimate of the fraction of zero precipitation by SPF and IK compared to the observed fraction of zeros (estimated as the proportion of stations that actually observed zero precipitation). Estimates of the fraction of wet area were not carried out for SeNorge. The estimate of p by SPF is scattered around the line representing a perfect match between modelled and observed values. For both the areas, the estimates of p by IK tend to underestimate coverage for events where less than half of the area is covered by precipitation and overestimate coverage for events where more than half of the area is covered by precipitation.

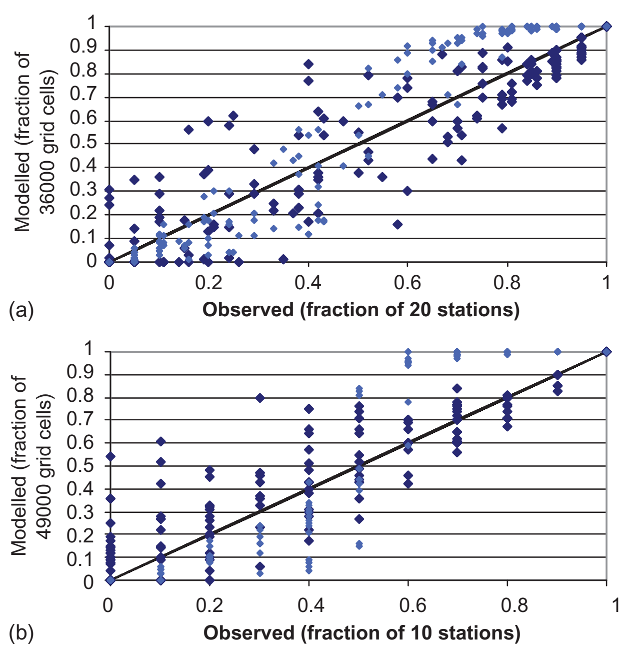

Fig. 10 Modelled and observed and spatial fraction of zeros for (a) the Oslo area, and (b) the Trondheim area. Small diamonds indicate result from IK, whereas large diamonds indicate those from SPF.

DISCUSSION

The increased use of spatially-distributed hydrological models also increases the demand on the quality of the interpolated meteorological fields used as input. If the interpolation method applied to the observed values produces too smooth a field, it undermines the very idea of distributed models and the motivation for using them. In principle, the more the spatial variability of the input is underestimated, the more the distributed models will tend to provide the same information as lumped models with spatial means as input. One could argue that the degree of distribution in space of the hydrological model should be carefully matched by the detail of the spatial distribution of the meteorological input. If the spatial variability of precipitation is systematically underestimated, one cannot expect to properly calibrate or model the different (nonlinear) hydrological response mechanisms. An informative data set is one which contains enough variability in watershed behaviour that the different modes of operation of the hydrological processes are properly represented (Sorooshian, 1995, p. 49). Also, when predicting hydrological response, it is better to have extreme precipitation estimated for the wrong location than not estimated at all. The latter will be the case if spatial variability is underestimated.

It is clear from Figs 7 and 8(b) that SPF is better at reproducing the observed spatial variability and that this feature is seriously underestimated by IK. The fact that the spatial variability of SPF is generally higher than IK could have had a negative impact on the precision in point estimation of SPF. The generally higher spatial variability of SPF clearly gives a higher potential for errors than IK with its very conservative estimate, since SPF provides a larger range of variation around the spatial mean than IK. The SPF method is thus more dependent on a good procedure for placing the estimated values correctly in space, since, due to this method's higher spatial variability, there is a greater risk of large deviations from the true precipitation values. It is surprising that the quite significant difference in estimate of spatial variance is not, somehow, reflected in the cross-validation results. Although RMSE and MAE for the Oslo area are slightly better for IK, the results for the other error measures, and for the Trondheim area, are very similar. One possible explanation may be that the data records of precipitation (one year) are not sufficient to resolve the, perhaps, subtle differences.

The cross-validation results for the extreme values show that the two methods perform similarly for the Oslo area and that SPF is somewhat better for the Trondheim area. The analysis contains few data but the comparison of extreme values is more in favour of SPF than when assessing the one-year data sample. Perhaps one can see here the effect of a more realistic estimate of spatial variance of SPF, in that one would expect significant errors in the estimates of extreme values if the spatial variance were underestimated. Analysis of longer time series is needed for further assessments to be made.

The derived algorithm for estimating the dry areas appears to work well. The agreement with the Rissa weather radar of the Trondheim area is very good. The differences between the estimates of intermittency by IK and SPF methods are indistinguishable when assessed for the cross-validation, and the systematic errors of under-/over-estimation to that of IK only appears when assessing the spatial data.

It can be argued that in order to employ SPF and kriging, an assumption of second-order stationarity in space (see Delhomme, 1978) has to be made. Whether SPF is subject to such constraints is not entirely clear in that the shape of the spatial distribution is allowed to vary for each event. In addition, the spatial distribution will, for some events, have a frequency of zeros. Stationarity of the second order for the precipitation fields from SPF is therefore not obvious and perhaps not even desired. The ability to produce non-stationary sequences is considered as a positive feature in rainfall modelling, and the application of generalized linear models on rainfall modelling problems has this feature (Chandler & Wheater, 2002). Nevertheless, implemented operationally, SPF will be employed for limited areas of 70×70 km^2, in an attempt to ensure some degree of spatial stationarity.

The semivariogram used in the kriging procedure defines the spatial structure (i.e. spatial auto-correlation) of the precipitation fields obtained by SPF. The type of the semivariogram is fixed as exponential and the sill is estimated, for each event, from the observed spatial variance. By using a fixed-type semivariogram, the spatial structure of the fields from SPF is unrealistic and probably too smooth. The spatial auto-correlation structure of precipitation fields is probably not well described for shorter time scales by the sort of climatological variograms (Lebel *et al*., 1987) employed in this study, and ideally, the spatial structure of each event should be determined. However, an analytical representation of the observed spatial structure is often difficult to determine and if SPF is used operationally, nationwide, the number of stations will often be too few for a meaningful estimate of spatial structure.

The SPF method is not an exact interpolator, which means that the interpolated precipitation field does not go through the observations. In contrast, kriging with zero nugget effect and inverse distance interpolation both do have this feature. The practical implication of SPF not being an exact interpolator is primarily that the observation points are not as well estimated as would be the case using kriging. The results of cross-validation, however, indicate that all the other points in the area of interest are estimated with a quality comparable to IK. As the number of observation points comprises a very tiny fraction of the area to be interpolated, the hydrological impact of SPF not being an exact interpolator is believed to be modest.

It is reasonable to assume that since the spatial variability is better estimated by SPF, the spatial distribution of precipitation obtained by SPF should be closer to reality than that of IK. In principle, there is thus a potential for SPF to provide better precipitation fields, and this serves as an inspiration to deriving better ways of distributing the simulated values in space than the method employed in this study. This line of enquiry will be followed in future work.

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